Endogenous Technological Change Along the Demographic Transition Online Supplement

Markus Pettersson[†]

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This supplement provides (a) derivations of the balanced growth rates in the main model; (b) details on the estimation of the life-cycle productivity profile; (c) details on the calibration of taxes; (d) additional sensitivity analyses; and (e) additional figures.

Appendix A Deriving the Stationary Growth Rates

The stationary growth rates of the benchmark model and of the version with labour as the only R&D input are straightforwardly derived following Jones (1995) and Boppart and Krusell (2020) as follows.

A.1 TFP, Output Per Capita, Hours Per Worker

In a stationary equilibrium, interior solutions to the household maximisation problem are characterised by an Euler equation, an intratemporal first-order condition, and a budget constraint of the forms

$$c_j^{-\sigma} = \beta s_j \left(1 + r(1-\tau^k)\right) \frac{\Omega_{j+1}}{\Omega_j} \mathbb{E}\left[c_{j+1}^{-\sigma} \,\middle|\, \eta \right], \tag{A.1}$$

$$\psi h_j^{1/\theta} = \Omega_j \frac{(1-\tau^{wm}(w\ell_j) - \tau^b) w \varepsilon_j \eta}{c_j^{\sigma}(1+\tau^c)}, \tag{A.2}$$

$$a_{j+1} + (1+\tau^c)c_j = \left(1 + r(1-\tau^k)\right)a_j + \left(1 - \tau^w(w\ell_j) - \tau^b\right)w\ell_j + tr + b(R_j), \tag{A.3}$$

where τ^{wm} in (A.2) denotes the *marginal* income tax rate. Let g_x denote the stationary growth rate of a variable x. Wages are standard neoclassical, so in a stationary equilibrium with a constant interest rate and capital-output ratio, wages grow by the rate of TFP. For the first-order condition (A.2) to hold along a balanced growth path, we then necessarily need $(1+g_h)^{1/\theta} = (1+g_Z)(1+g_c)^{-\sigma}$. Likewise, the budget constraint (A.3) is only consistent with balanced growth if consumption grows by the rate of output per capita and labour income: $1+g_c = 1+g_y = (1+g_Z)(1+g_h)$. Meanwhile, TFP is just the measure of intermediate firms raised to a power, $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$, so its growth rate is similarly the growth rate of

[†] Department of Economics, Stockholm University. Email: markus.pettersson@su.se.

intermediate firms raised to the same power. Combining these conditions yields that the growth rates of TFP, output per capita, and hours per worker must satisfy

$$1 + g_Z = (1 + g_z)^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}, \qquad 1 + g_y = (1 + g_Z)^{\frac{1+\theta}{1+\theta\sigma}}, \qquad 1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}, \tag{A.4}$$

and substituting $1 + g_z = (1 + n)^\chi$ for some χ into Equation (A.4) gives the growth rates in Definition 2 in the main paper.

A.2 Intermediate-Firm Growth Rate: Baseline Model

Next, the intermediate-firm growth rate is given by $1 + g_z = 1 - \delta_z + \nu Q^\lambda z^{\phi-1}$, and this is constant if and only if the last term on the right-hand side is constant. The latter holds only if

$$(1 + g_z)^{1-\phi} = (1 + g_Q)^\lambda. \tag{A.5}$$

The growth rate of R&D investment equals the aggregate output growth rate $(1 + g_Z)(1 + g_L)$ by the goods market condition. Employment grows by the rate of the population $1 + n$ whereas labour productivity is constant for a fixed population age structure. The labour force growth rate is therefore given by $1 + g_L = (1 + g_h)(1 + n)$. Together with Equation (A.4), this allows us to rewrite the growth rate of R&D investment into

$$1 + g_Q = (1 + g_Z)(1 + g_h)(1 + n) = (1 + g_z)^{\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(1 + n).$$

Plugging this into (A.5) and rearranging terms yields the growth rate in Definition 2 in the main paper:

$$1 + g_z = (1 + n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}. \tag{A.6}$$

A.3 Intermediate-Firm Growth Rate: R&D With Only Labour

If R&D uses only labour, then the R&D process is given by $1 + g_z = 1 - \delta_z + \nu L_z^\lambda z^{\phi-1}$. In a stationary equilibrium, R&D labour L_z must grow by the rate of total labour supply according to the labour market condition. Again inspecting the right-hand side, we thus get a constant growth rate $1 + g_z$ if and only if

$$(1 + g_z)^{1-\phi} = (1 + g_L)^\lambda. \tag{A.7}$$

Using Equation (A.4), we can rewrite the labour force growth rate into

$$1 + g_L = (1 + g_h)(1 + n) = (1 + g_z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(1 + n).$$

Plugging this into (A.7) and rearranging terms yields the growth rate

$$1 + g_z = (1 + n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}. \tag{A.8}$$

Note that TFP growth collapses to the benchmark growth rate in Jones (1995) if we consider a steady state with constant hours worked (via log preferences, $\sigma \to 1$) and a substitution parameter for intermediate goods exactly equal to the capital share parameter ($\rho = \alpha$).

Appendix B Estimating the Life-Cycle Earnings Profile

I parametrise the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ as the exponential of a quadratic age polynomial: $\varepsilon_j = \exp\left\{\vartheta_0 + \vartheta_1 j + \vartheta_2 j^2\right\}$. In the model, the hourly wage of an individual i of age j at time t is given by $w_{ijt} = w_t \varepsilon_j \eta_{ijt}$, where w_t is a common wage trend and η_{ijt} captures any idiosyncratic differences. This motivates the fixed effects regression

$$\ln w_{ijt} = \varrho_t + \varrho_i + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2 + u_{ijt}, \tag{B.1}$$

where ϱ_t is a time fixed effect, ϱ_i is an individual fixed effect, and u_{ijt} is an error term. Equation (B.1) implicitly captures cohort effects through the individual fixed effects and it is well known that collinearity between age, time, and cohorts prohibits simultaneous identification of these effects. As a partial remedy, I use the approach advocated by Heckman and Robb (1985) and replace the time fixed effect by two macroeconomic variables which plausibly proxy for the underlying unobserved time variables in the context of an earnings regression: log of the aggregate real wage level and the percentage point deviation of the unemployment rate from its long-run mean. The former corresponds to $\ln w_t$ and controls for secular wage growth and the latter (which is also used by French, 2005) controls for fluctuations in the business cycle.

I estimate Equation (B.1) with micro data on earnings from the nationally representative SRC sample of the Panel Study of Income Dynamics (PSID) for survey years 1968 to 2019 (which correspond to calendar years 1967 to 2018). Individual wages are imputed as total annual labour income divided by annual hours worked. The aggregate wage used to proxy the time fixed effect is obtained from the national accounts by dividing total private industry wages (BEA NIPA Table 2.3) by total private industry hours worked (BEA NIPA Table 6.9). The unemployment rate is taken from the Bureau of Labor Statistics (BLS, series ID LNS14000000). All nominal variables are deflated into 2012 dollars using the PCE price index (BEA NIPA Table 2.3.4).

For the benchmark estimation, I impose standard sample restrictions (see for instance French, 2005, Heathcote, Storesletten and Violante, 2010, and Huggett, Ventura and Yaron, 2011): I select male household heads with no inconsistencies in reported age, who work between 300 and 5,840 hours a year (30 percent of part time and twice full time, respectively), and whose hourly wage exceeds \$3 per hour and does not exceed \$100 per hour in 2012 dollars. I consider individuals between the ages of 18 and 75. This goes against the standard practice of excluding ages at the beginning and end of the working life to avoid sample selection issues relating to work-life entry and exit. This choice is motivated by the need for an efficiency profile for all ages above 20, given that retirement in the model is endogenous. The alternative, estimating the age profile on individuals between, say, the ages of 25 and 60, instead requires extrapolation of the age profile to younger and older ages, and it is not clear that this approach is preferable. An upper bound at 75 is nevertheless imposed to ensure there are at least 100 observations in each age group. Extrapolation beyond 75 is inconsequential, since between 95 and 99 percent of model households retire before 75. The final sample consists of 90,832 person-year observations.

Table B.1 shows the estimation results along with several robustness checks. Column (1) corresponds to the age profile in Figure 4a in the paper. Column (2) shows standard OLS estimates and column (3) includes only individual fixed effects. In both cases, secular wage growth is interpreted as life-cycle earnings differences, and this generates steeper profiles; productivity at peak age is 115 to 130 percent larger than the initial age, compared to 70 percent for the main estimation. This underlines the importance

Table B.1. Estimation of deterministic age-efficiency profile.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	-1.3734^{***} (0.2250)	0.9857^{***} (0.0191)	1.1419^{***} (0.0365)	-0.9102^{***} (0.1783)	-1.2920^{***} (0.2043)	-1.0902^{***} (0.1655)	-1.1382^{***} (0.1045)
ϑ_1	0.0734^{***} (0.0020)	0.0954^{***} (0.0010)	0.0835^{***} (0.0018)	0.0813^{***} (0.0018)	0.0709^{***} (0.0018)	0.0610^{***} (0.0015)	0.0606^{***} (0.0010)
ϑ_2	-0.0008^{***} (0.0000)	-0.0010^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0007^{***} (0.0000)	-0.0007^{***} (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.160	0.121	0.156	0.162	0.158	0.152	0.153

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.
[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).
[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

of controlling for time effects. Column (4) changes the aggregate wage measure to average hourly earnings of production and nonsupervisory employees (BLS, series ID CES0500000008). This wage exhibits lower growth in recent decades than the imputed wage from the BEA, and the estimation results are therefore similar to columns (2) and (3).[1] Columns (5) to (7) expand the sample to include spouses, partners, and female household heads and add additional individual-level controls that may change over time. These additions lower the point estimates somewhat, although these samples raise additional concerns for sample selection and also provide worse fits to the data as measured by the adjusted R^2.

Lastly, for the model scenario in which wages are an increasing function of hours worked in Appendix D, I run identical estimations to above with the only exception that individual wages are constructed as (total annual labour income)/(annual hours worked)$^{1.415}$ in the PSID data. Table B.2 shows the results from these regressions.

Appendix C Constructing Tax Rates

This section explains the calibration of the model taxes. First, I describe the construction of the aggregate tax rates from the national accounts, then the estimation a progressive income tax function, and finally the calibration of the income tax function used in the model (which builds on the former two).

[1] I use the BEA wage measure as the benchmark since the BLS wage is more limited in scope.

Table B.2. Estimation of deterministic age-efficiency profile with part-time wage penalty.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	-4.1083^{***} (0.2291)	-1.8276^{***} (0.0200)	-1.6727^{***} (0.0366)	-3.6135^{***} (0.1809)	-4.0415^{***} (0.2084)	-3.9382^{***} (0.1669)	-3.9195^{***} (0.1121)
ϑ_1	0.0540^{***} (0.0020)	0.0757^{***} (0.0010)	0.0634^{***} (0.0018)	0.0616^{***} (0.0018)	0.0515^{***} (0.0018)	0.0438^{***} (0.0014)	0.0400^{***} (0.0010)
ϑ_2	-0.0006^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0005^{***} (0.0000)	-0.0004^{***} (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.123	0.094	0.120	0.125	0.120	0.110	0.113

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours$^{1.415}$. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.
[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).
[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

C.1 Aggregate Tax Rates

The methodology to construct the aggregate tax rates on consumption, capital, and labour income is taken off-the-shelf from Fernández-Villaverde *et al.* (2015), which in turn builds on Jones (2002) and Mendoza, Razin and Tesar (1994). In short, each tax rate is given by aggregating all relevant tax revenues at the general government level and then dividing by the corresponding tax base. All data for this exercise are taken from the BEA NIPA tables. Table C.1 summarises the variables that I use.

The average consumption tax rate τ^c is given by

$$\tau^c = \frac{TPI - PRT}{C - (TPI - PRT)}. \tag{C.1}$$

The numerator of (C.1) is the revenue from consumption taxation. I subtract property taxes from total taxes on production because homeowners in the national accounts are treated as businesses that rent their properties to themselves. Property taxes are therefore incorporated as taxes on capital instead. The consumption tax base in the denominator is total personal consumption expenditures net of consumption taxes paid (that is, the pre-tax value of consumption).

Table C.1. Tax data variables.

Variable	Explanation	Source	
C	Personal consumption expenditures	BEA NIPA Table 1.1.5	line 2
EC	Compensation of employees	BEA NIPA Table 1.12	line 2
W	Wages and salaries	BEA NIPA Table 1.12	line 3
PRI	Proprietors' income[a]	BEA NIPA Table 1.12	line 9
RI	Rental income of persons[a]	BEA NIPA Table 1.12	line 12
CP	Corporate profits[a]	BEA NIPA Table 1.12	line 13
NI	Net interest and miscellaneous payments	BEA NIPA Table 1.12	line 18
PCT	Personal current taxes	BEA NIPA Table 3.1	line 3
TPI	Taxes on production and imports	BEA NIPA Table 3.1	line 4
CT	Taxes on corporate income	BEA NIPA Table 3.1	line 5
CSI	Contributions for government social insurance	BEA NIPA Table 3.1	line 7
PRT	Property taxes	BEA NIPA Table 3.3	line 9

[a] With inventory valuation adjustment and capital consumption adjustment.

The NIPA tables do not provide a breakdown of personal current taxes into labour and capital income. To make this split, I construct an average personal income tax rate τ^p as an intermediate step via

$$\tau^p = \frac{PCT}{W + PRI/2 + CI}, \qquad \text{where} \qquad CI \equiv PRI/2 + RI + CP + NI.$$

The numerator is the sum of personal current taxes at the federal, state and local levels. The tax base is the sum of wages, proprietors' income, and capital income (CI). Here, proprietors' income is divided evenly between labour and capital income following Jones (2002), who emphasises that any split into labour and capital income is arbitrary and therefore chooses the fifty-fifty split as a middle ground.

I then estimate the total revenue from personal taxes on income and capital as $\tau^p(W + PRI/2)$ and $\tau^p CI$, respectively. The average labour income and capital tax rates are subsequently given by

$$\tau^w = \tau^p \frac{W + PRI/2}{EC + PRI/2} \qquad \text{and} \qquad \tau^k = \frac{\tau^p CI + CT + PRT}{CI + PRT}. \tag{C.2}$$

One robustness check in Appendix D also considers an exogenous social security contribution rate τ^b. For this scenario, I construct the social security tax rate as

$$\tau^b = \frac{CSI}{EC + PRI/2}. \tag{C.3}$$

The sum of τ^w and τ^b gives the measure of the average labour income tax rate used by Fernández-Villaverde *et al.* (2015). Figure C.1 plots the estimated tax rates, which highlights that imposing constant tax rates in the model for consumption, labour income and capital is a reasonable assumption.

C.2 Estimating the Income Tax Rate Function

To estimate the income tax function, I compute average tax rates at hypothetical levels of income and fit Equation (35) in the main paper to these synthetic data. This process follows the methodology of the

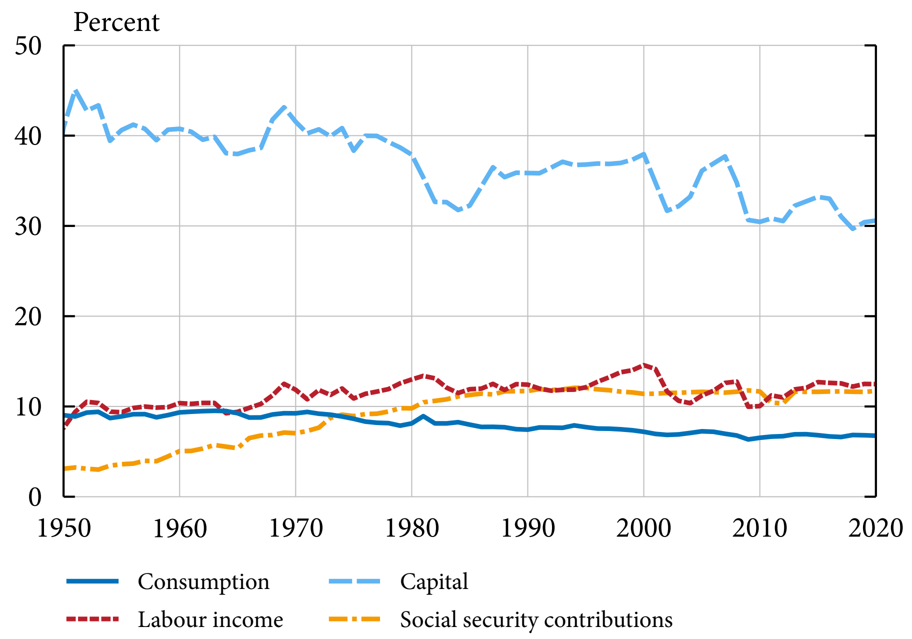

Figure C.1. Aggregate tax rate estimates from the national accounts.

OECD tax database for the United States, which creates effective tax rates by applying applicable tax rules and regulations for given years and earnings levels and then dividing the resulting net tax liabilities by gross earnings. These calculations include taxation at all levels of government for a household assumed to live in Detroit, Michigan.

For simplicity, I only consider single households without children, in line with the primary estimates published by the OECD.[2] This choice is of secondary importance since I eventually scale the estimated tax function to match the national accounts. I also abstract from social security contributions since those are modelled separately in my framework. The subsections below outline the formulas, parameters, and parameter values for this particular case.[3]

C.2.1 Taxable Earnings

Taxable earnings at government level $\mathtt{x} \in \{\mathtt{fed}, \mathtt{state}, \mathtt{local}\}$ is given by gross income GI minus a tax allowance $\mathrm{TAXALLOW}_{\mathtt{x}}$, provided that this is positive:

$$e^{\mathtt{x}}(\mathrm{GI}) \;=\; \max\{\mathrm{GI} - \mathrm{TAXALLOW}_{\mathtt{x}},\, 0\}.$$

At the federal level, the allowance consists of a standard deduction STDALLOW and a personal exemption $\mathrm{EXEMPT}_{\mathtt{fed}}$. The personal exemption is reduced at a taper rate $\varphi^T_{\mathtt{ex}}$ for every USD 2,500 that gross income exceeds the threshold $\mathrm{THOLD}_{\mathtt{ex}}$. At the state and local levels, the allowances are fixed personal exemptions $\mathrm{EXEMPT}_{\mathtt{state}}$ and $\mathrm{EXEMPT}_{\mathtt{local}}$, respectively. Thus,

$$\mathrm{TAXALLOW}_{\mathtt{fed}} \;=\; \mathrm{STDALLOW} \;+\; \mathrm{EXEMPT}_{\mathtt{fed}} \left(1 - \varphi^T_{\mathtt{ex}} \left\lceil \frac{\max\{\mathrm{GI} - \mathrm{THOLD}_{\mathtt{ex}},\, 0\}}{2500} \right\rceil \right),$$

[2] See Table I.5, available for download at OECD.Stat.

[3] The implementation code (available upon request) also incorporates different household compositions (with respect to children and marital status) and social security contributions. Supplementary documentation for these cases is given in the OECD publication *Taxing Wages* available at the OECD iLibrary.

$$\text{TAXALLOW}_{\text{state}} = \text{EXEMPT}_{\text{state}},$$

$$\text{TAXALLOW}_{\text{local}} = \text{EXEMPT}_{\text{local}},$$

where $\lceil \cdot \rceil$ is the ceiling function: $\lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$.

C.2.2 Taxes

Taxable earnings at the state and local levels are subject to flat tax rates τ^{state} and τ^{local}. The federal income tax is progressive, with higher marginal tax rates at higher levels of income. Consider N federal tax brackets with marginal tax rates $\tau_1^{\text{fed}}, \ldots, \tau_N^{\text{fed}}$ starting at earnings thresholds $\bar{e}_1, \ldots, \bar{e}_N$, where $\bar{e}_1 = 0$. Given a largest applicable federal tax bracket $I = \max\{i : e^{\text{fed}}(\text{GI}) > \bar{e}_i\}$, the tax liability at each level of government is then given by functions $T^{\text{x}}(\text{GI})$ of gross income as follows:

$$T^{\text{fed}}(\text{GI}) = \sum_{i=1}^{I-1} \tau_i^{\text{fed}} \left(\bar{e}_{i+1} - \bar{e}_i\right) + \tau_I^{\text{fed}} \left(e^{\text{fed}}(\text{GI}) - \bar{e}_I\right),$$

$$T^{\text{state}}(\text{GI}) = \tau^{\text{state}}\, e^{\text{state}}(\text{GI}),$$

$$T^{\text{local}}(\text{GI}) = \tau^{\text{local}}\, e^{\text{local}}(\text{GI}).$$

C.2.3 Tax Credits

The OECD considers two types of federal tax credits for households without children: the Earned Income Tax Credit (EIC) and the Making Work Pay tax credit (MWP). The EIC and the MWP provide refundable tax credits equal to some fractions φ_{eic} and φ_{mwp} of gross income up to some maximum amounts $\varphi_{\text{eic}}\,\overline{\text{eic}}$ and $\overline{\text{mwp}}$. The tax credits are phased out at taper rates φ_{eic}^T and φ_{mwp}^T once gross income exceeds thresholds $\text{THOLD}_{\text{eic}}$ and $\text{THOLD}_{\text{mwp}}$. The total tax credit amounts from these programs are thus given by

$$\text{eic}(\text{GI}) = \max\left\{\varphi_{\text{eic}} \min\left\{\text{GI}, \overline{\text{eic}}\right\} - \varphi_{\text{eic}}^T \max\left\{\text{GI} - \text{THOLD}_{\text{eic}}, 0\right\}, 0\right\}$$

and

$$\text{mwp}(\text{GI}) = \max\left\{\min\left\{\varphi_{\text{mwp}}\text{GI}, \overline{\text{mwp}}\right\} - \varphi_{\text{mwp}}^T \max\left\{\text{GI} - \text{THOLD}_{\text{mwp}}, 0\right\}, 0\right\}.$$

Total federal tax credits is the sum of EIC and MWP. At the state level, the OECD includes the Michigan Earned Income Tax Credit, which is an additional refundable credit equal to a fraction φ_{meic} of the federal EIC amount. The local level incorporates the Michigan City Income Tax Credit (CTC) which is a nonrefundable credit equal to some fraction of the total local tax liability $T^{\text{local}}(\text{GI})$ up to some maximum amount $\overline{\text{ctc}}$. Below this upper bound, the CTC credit rates decline with income. Consider N credit rate brackets with marginal credit rates $\varphi_{1,\text{ctc}}, \ldots, \varphi_{N,\text{ctc}}$ starting at tax liability thresholds $\overline{T}_1, \ldots, \overline{T}_N$, where $\overline{T}_1 = 0$. Given a largest applicable tax credit bracket $I = \max\{i : T^{\text{local}}(\text{GI}) > \overline{T}_i\}$, the total tax credit at each level of government is then given by functions $C^{\text{x}}(\text{GI})$ of gross income as follows:

$$C^{\text{fed}}(\text{GI}) = \text{eic}(\text{GI}) + \text{mwp}(\text{GI}),$$

$$C^{\text{state}}(\text{GI}) = \varphi_{\text{meic}}\,\texttt{eic}(\text{GI}),$$

$$C^{\text{local}}(\text{GI}) = \min\left\{\sum_{i=1}^{I-1}\varphi_{i,\text{ctc}}\left(\overline{T}_{i+1} - \overline{T}_i\right) + \varphi_{I,\text{ctc}}\left(T^{\text{local}}(\text{GI}) - \overline{T}_I\right),\ \overline{\texttt{ctc}}\right\}.$$

C.2.4 Effective Income Tax Rate

The effective income tax rate $\tau^w(\text{GI})$ at gross income GI is the total tax liability net of tax credits measured as a percentage of gross income:

$$\tau^w(\text{GI}) = \frac{1}{\text{GI}}\sum_{\text{x}\in X}\left(T^{\text{x}}(\text{GI}) - C^{\text{x}}(\text{GI})\right),$$

where $X = \{\texttt{fed}, \texttt{state}, \texttt{local}\}$. In the practical implementation of these tax calculations, we consider an average gross income level $\overline{\text{GI}}$ and then express all other gross incomes as a percentage of that average.

C.2.5 Estimation

Using the methodology above, I create effective income tax rates on a grid of gross incomes for each year between 2000 and 2022. The grid is linearly spaced with 401 points, ranging from 0 to a multiple 20 of average gross income. The parameter values for this exercise are collected from the OECD and are available upon request. I then fit the income tax function $\tau^w(\text{GI}) = \kappa_0\left[1 - \left(\kappa_2\left(\frac{\text{GI}}{\overline{\text{GI}}}\right)^{\kappa_1} + 1\right)^{-\frac{1}{\kappa_1}}\right] + \kappa_3$ by a nonlinear OLS to these tax rates.

Figure C.2 shows the constructed tax rates for the lower end of the income grid together with the corresponding fit and its estimated coefficients. Even though the period considered saw two major tax reforms (the Economic Growth and Tax Reconciliation Relief Act of 2001 and the Tax Cuts and Jobs Act of 2017) and underwent three economic downturns (the early 2000s recession, the Great Recession, and the COVID-19 recession), effective income tax rates remain largely stable over time. Therefore, the estimated tax function provides a close fit of the constructed tax rates, as seen by the high R^2 of 0.97.

C.3 Changing the Tax Rate Level While Maintaining Progressivity

Once the income tax function is estimated, I adjust its level so that the tax rate at average earnings matches the tax rate from the national accounts. To this end, I follow Guvenen, Kuruscu and Ozkan (2014) to ensure that the degree of progressivity remains the same before and after. Thus, let $\widetilde{\tau}(e)$ be some average tax rate function of the Gouveia and Strauss (1994) form:

$$\widetilde{\tau}(e) = \widetilde{\kappa}_0\left[1 - \left(\widetilde{\kappa}_2\left(\frac{e}{\overline{e}}\right)^{\widetilde{\kappa}_1} + 1\right)^{-\frac{1}{\widetilde{\kappa}_1}}\right] + \widetilde{\kappa}_3. \tag{C.4}$$

Denote its corresponding marginal tax rate by $\widetilde{\tau}^m(e) = \frac{\partial}{\partial e}\left(\widetilde{\tau}(e)e\right)$. To change the level of this tax function into a similar function $\tau(e)$ with parameters $\kappa_0, \ldots, \kappa_3$ without changing the its progressivity, we need the

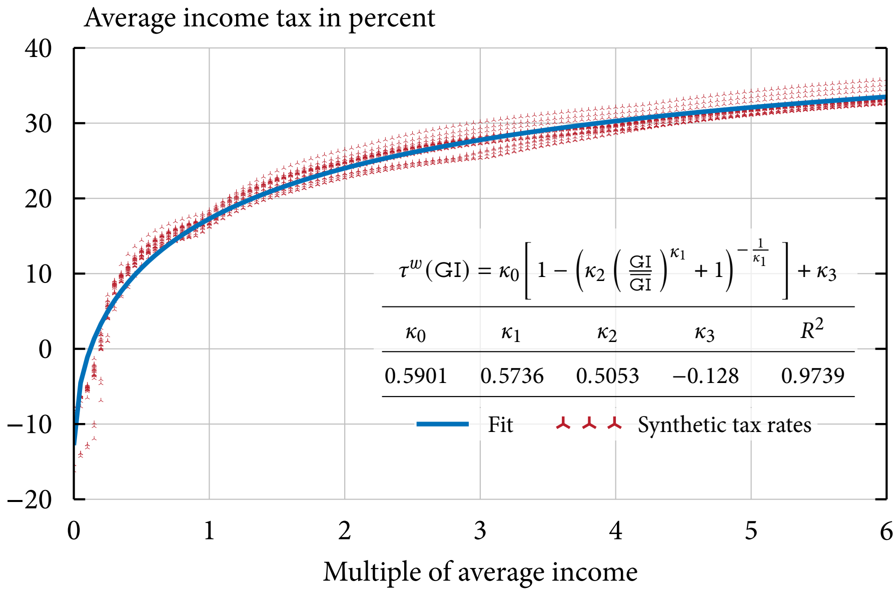

Figure C.2. Estimation of the income tax function.

ratio of net take-home shares at any two earnings levels e and e' to be the same in both tax systems:

$$\frac{1 - \tau^m(e')}{1 - \tau^m(e)} = \frac{1 - \widetilde{\tau}^m(e')}{1 - \widetilde{\tau}^m(e)}.$$

This expression can be rearranged to obtain

$$\tau^m(e) = 1 - \bar{k}\big(1 - \widetilde{\tau}^m(e)\big), \qquad \text{where} \qquad \bar{k} \equiv \frac{1 - \tau^m(e')}{1 - \widetilde{\tau}^m(e')} \tag{C.5}$$

is a level ratio between the two tax systems that we are free to choose. Since $\tau(e)e = \int_0^e \tau^m(x)\,dx$, we can integrate Equation (C.5) to obtain an average tax rate of a similar form:

$$\tau(e) = 1 - \bar{k}\big(1 - \widetilde{\tau}(e)\big). \tag{C.6}$$

Substituting Equation (C.4) into (C.6) and rearranging terms, we finally get

$$\tau(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3,$$

where $\kappa_0 \equiv \bar{k} \cdot \widetilde{\kappa}_0$, $\kappa_1 \equiv \widetilde{\kappa}_1$, $\kappa_2 \equiv \widetilde{\kappa}_2$ and $\kappa_3 \equiv 1 - \bar{k}(1 - \widetilde{\kappa}_3)$. The calibrated $\kappa_0, \ldots, \kappa_3$ in the benchmark model use the estimates in Figure C.2 as $\widetilde{\kappa}_0, \ldots, \widetilde{\kappa}_3$ and set the scale parameter $\bar{k}$ such that the tax rate at average income $\bar{e}$ matches the income tax rate τ^{NA} from the national accounts. Specifically, the latter requires that $\tau(\bar{e}) = \tau^{NA} = 1 - \bar{k}\big(1 - \widetilde{\tau}(\bar{e})\big)$, which can be rearranged to give the scale parameter as

$$\bar{k} = \frac{1 - \tau^{NA}}{1 - \widetilde{\tau}(\bar{e})}, \qquad \text{where} \qquad \widetilde{\tau}(\bar{e}) = \widetilde{\kappa}_0 \left[1 - \left(\widetilde{\kappa}_2 + 1 \right)^{-\frac{1}{\widetilde{\kappa}_1}} \right] + \widetilde{\kappa}_3$$

is a function of estimated parameters only.

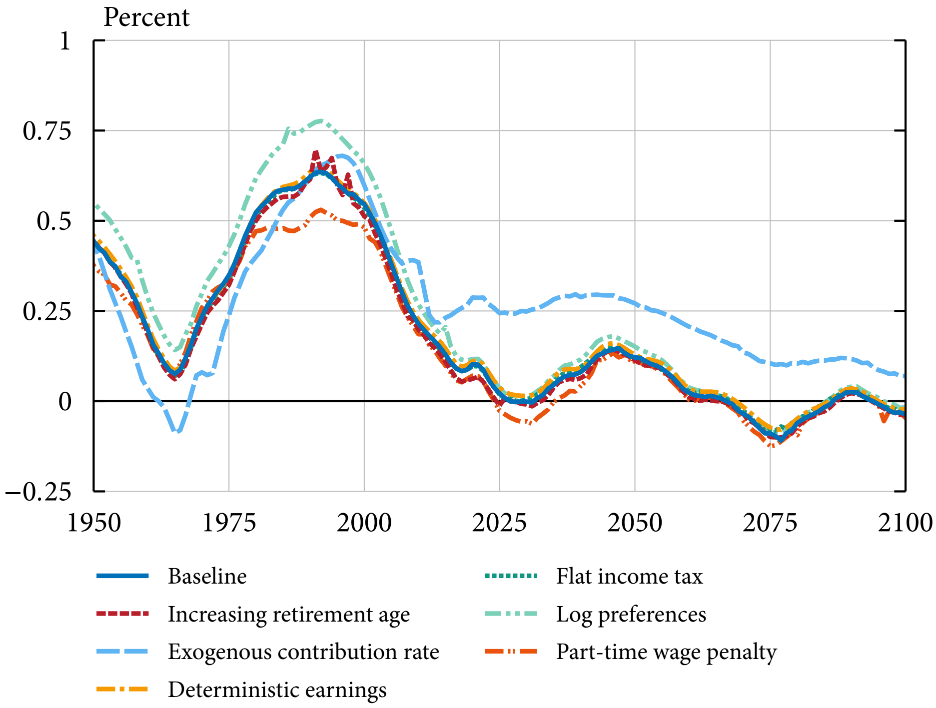

Figure D.1. Sensitivity check: growth rates with different model specifications.

Appendix D Additional Sensitivity Analyses

This appendix complements Section 7 in the main paper with additional robustness checks. Each robustness scenario recalibrates the preference parameters β and ψ to match the same calibration targets as in the baseline if needed. The growth rate of each alternative is plotted against the baseline in Figure D.1 while Figure D.2 shows their growth decompositions. Overall, neither alternative alters any qualitative conclusion and only the case with a different pension system configuration has a quantitatively meaningful impact.

Increased retirement age. The social security system in the benchmark model has a fixed normal retirement age (NRA) and early/delayed pension scaling schedule, contrary to reality. Since the average age of retirement influences all growth mechanisms in the paper, I therefore consider an alternative which more accurately describes the NRA and the delayed retirement credits. Specifically, I increase the NRA to 66 for cohorts born between 1940 and 1956 and to 67 for subsequent cohorts. Moreover, the delayed retirement credit is increased by 0.5 percentage points for every other cohort between 1924 and 1943. That is, the delayed retirement credit is 3 percent for cohorts born before 1925, 3.5 percent for the 1925–1926 cohorts, . . . , 7.5 percent for the 1941–1942 cohorts, and 8 percent for all subsequent cohorts. Overall, these changes nevertheless leave the baseline results unaffected because most households in the baseline already retire between the ages of 65 and 70.

Exogenous contribution rate. The benchmark model balances the social security budget by changing the pension contribution rate to maintain a fixed replacement rate. The fiscal pressure of an ageing population makes this the most growth pessimistic setup of the pension system: it reduces the incentive (through higher pension income) and ability (through higher taxation) to save for retirement as well as the incentive for late retirement (through a higher opportunity cost of working once eligible for social security) compared to a case which maintains tax levels by cutting pensions. Here, I consider the more optimistic case with exogenous taxation and endogenous changes to the replacement rate. To this end, I

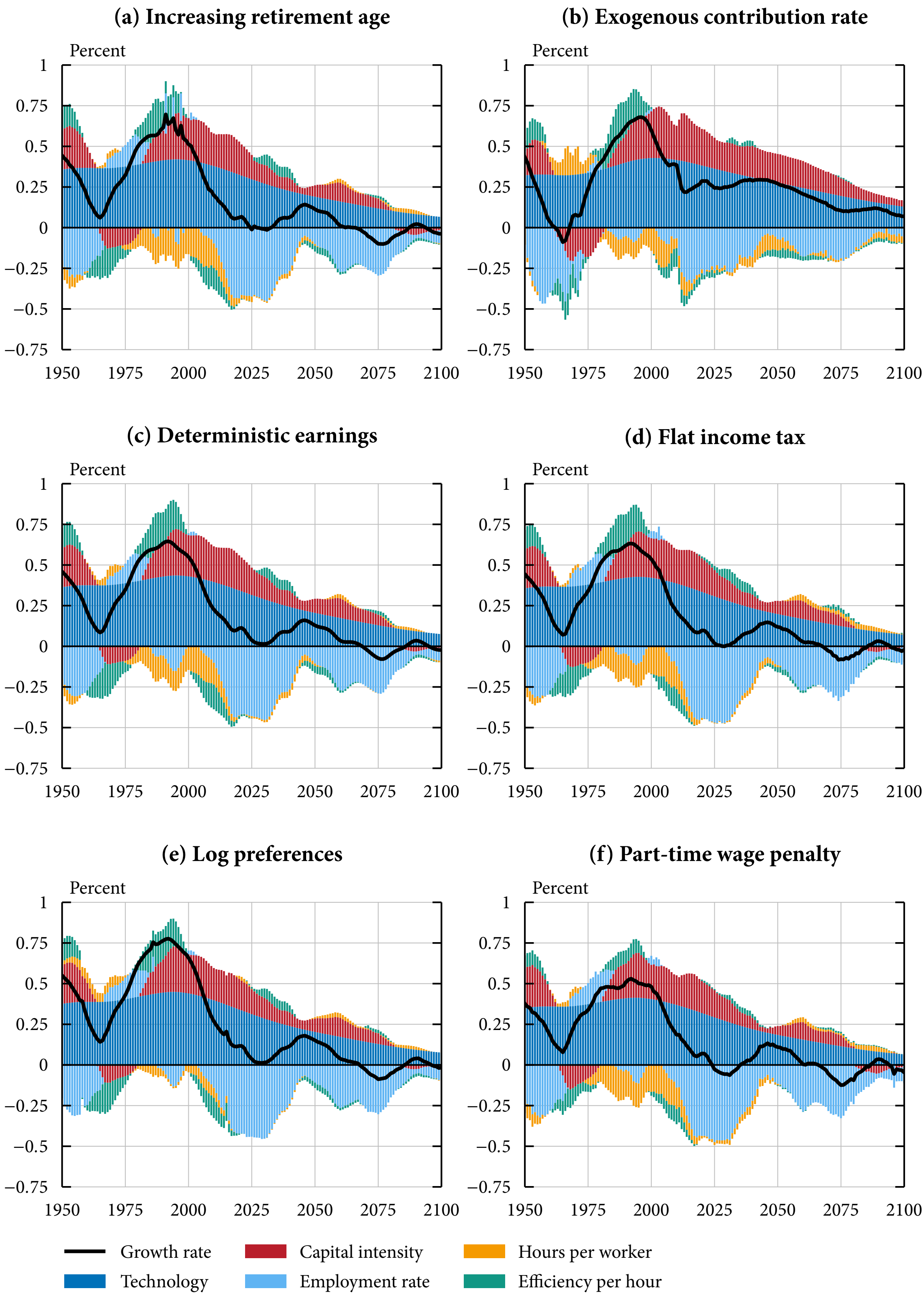

Figure D.2. Sensitivity check: growth decompositions with different model specifications.

construct a tax rate based on the national accounts measure for social insurance contributions (described in Appendix C) and feed this time series into the model.

Figure C.1 shows that the contribution rate rises throughout the post-war period. In the model, the resulting increase in contributions is more than sufficient to offset the increased pension bill of an ageing population, so benefits grow more generous, peaking at a replacement rate of 0.65 around 1990. After 1990, the contribution rate stabilises, causing the public sector to cut benefits throughout the twenty-first century. The increased generosity during the twentieth century reduces growth relative to baseline by 0.08 percentage points per year as households retire earlier and save less. Conversely, the decline in the replacement rates during the twenty-first century increases growth relative to the baseline by 0.16 percentage points per year.

Deterministic earnings. In the baseline, households face uninsurable idiosyncratic productivity shocks that add a savings motive beyond the standard life-cycle motive. In an alternative, I remove this feature and consider deterministic earnings. Although this reduces inequality and the overall level of household savings, the difference to the baseline turns out to be negligible.

Flat income tax. Rather than considering a progressive income tax, I analyse an alternative in which all households face a constant marginal (and average) income tax rate equal to that obtained from the national accounts: $\tau^w = 0.115$. Contrary to the deterministic earnings scenario, imposing a flat tax increases inequality and the level of household savings. Again, the difference to the baseline is nevertheless negligible.

Log preferences. Another nonstandard feature is that I consider preferences of the Boppart and Krusell (2020) class that generate declining hours worked along a balanced growth path. By contrast, a large part of the macroeconomic literature restricts itself to a subset of this class, defined by King, Plosser and Rebelo (1988), in which hours worked are constant in the long run. Constant long-run hours in my model are obtained as the special case when $\sigma \to 1$, so that flow utility becomes $u(c_j, h_j) = \Omega_j \log(c_j) - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}$. In this case, the income and substitution effect on leisure exactly offset each other, so hours worked do not fall when growth is positive. Between 1950 and 2000, annual growth with logarithmic preferences is therefore about 0.1 percentage points higher. This difference is explained entirely by the different adjustments in hours worked. Growth during the twenty-first century does not change since, with wage growth around zero, the response in hours is similar in both scenarios.

Part-time wage penalty. Several authors stress the importance of nonconvexities in the budget set to generate endogenous retirement (see for instance Rogerson and Wallenius, 2013, and Ljungqvist and Sargent, 2014). Social security plays this role in the baseline model. Another commonly used nonconvexity is nonlinear wages, which is motivated by the empirical observation that part-time work does not pay as high hourly wage as full-time work. Thus, following French (2005), consider household labour earnings given by $w\varepsilon_j \eta h_j^{1+\xi}$, with $\xi \geq 0$. The labour market condition then changes to $L = \sum_j \int_X \varepsilon_j \eta h_j(x)^{1+\xi} \, \mathrm{d}\Phi_j$ and, by similar derivations as in Appendix A, the long-run growth rate of intermediate firms becomes

$$1 + g_z = (1+n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma - \xi\theta(1-\sigma)} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}.$$

The long-run growth rates of output per capita and hours per worker similarly become

$$1 + g_y = (1 + g_Z)^{\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}} \qquad \text{and} \qquad 1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma-\xi\theta(1-\sigma)}}.$$

For $\xi = 0$, wages are a linear function of hours worked and we obtain the benchmark model. Here, I follow French and set the value of ξ to 0.415 based on Aaronson and French's (2004) empirical finding that a 50 percent reduction in hours corresponds to a 25 percent lower hourly wage. I recalibrate the model under the assumption that $\xi = 0.415$ holds, which lowers the intertemporal elasticity of substitution, $\sigma = 1.84$, and flattens the age-efficiency profile.[4]

With these adjustments, the growth rate is similar to the baseline on average but exhibits more stable dynamics during the twentieth century. The latter is due to two changes to the average productivity per hour worked. First, declining hours worked negatively impacts average productivity since the productivity of an individual worker, $\varepsilon_j \eta h_j^{\xi}$, now includes hours worked. Second, the age-efficiency profile is flatter than in the baseline. Changes in the age composition of the labour force, which are more prominent in the twentieth century, therefore leads to smaller changes in average efficiency.

[4] For the baseline, I estimate the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ from a PSID wage measure obtained by dividing annual labour income with annual hours. Here, I assume that $\xi = 0.415$ holds and construct PSID wages as $(\text{total annual labour income})/(\text{annual hours worked})^{1.415}$ (see Appendix B for estimation details).

Appendix E Additional Figures

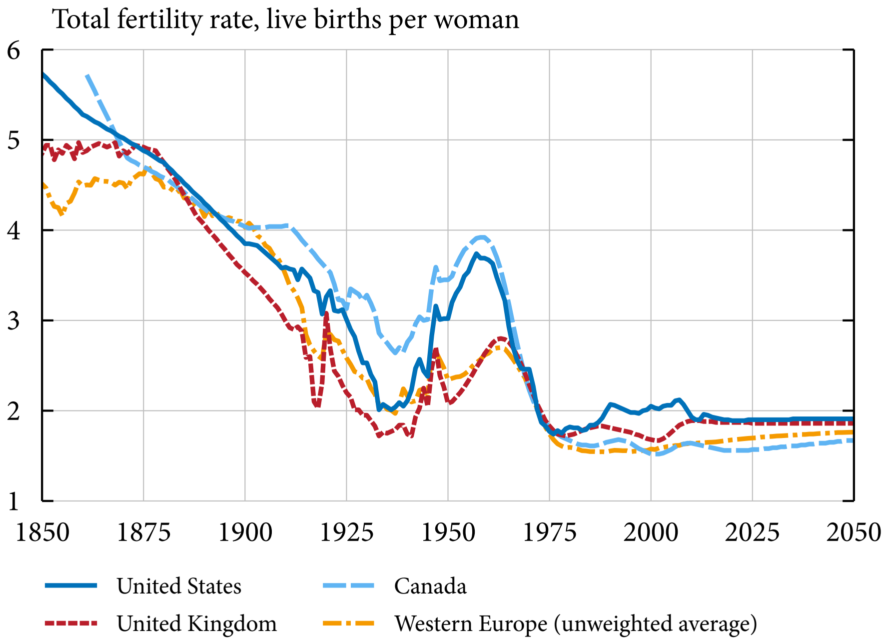

Figure E.1. Fertility across major economies.

Source. Gapminder Foundation (2017).

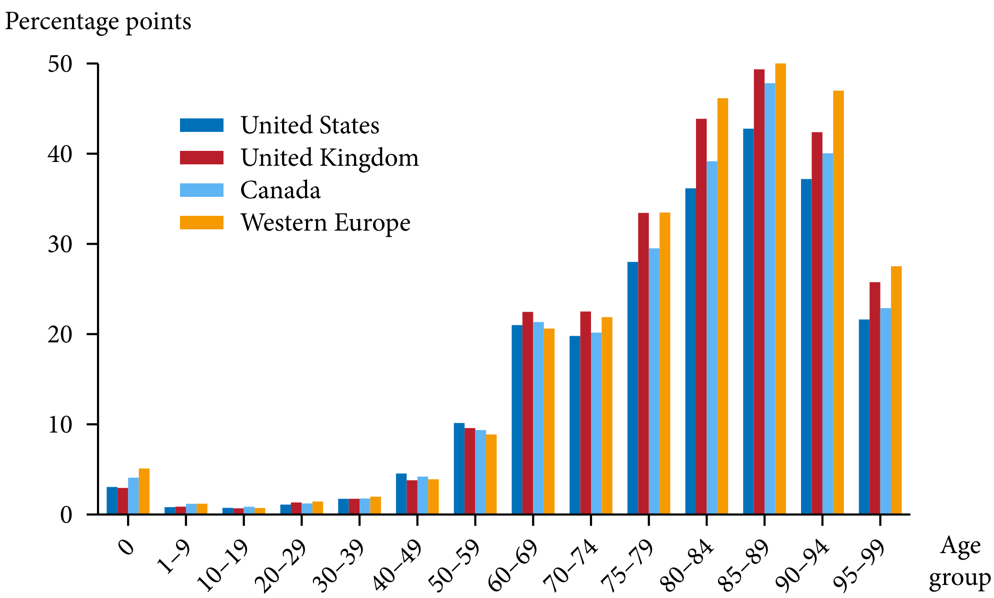

Figure E.2. Increase in survival probabilities by age group across major economies, 1950–2100.

Source. United Nations (2022).

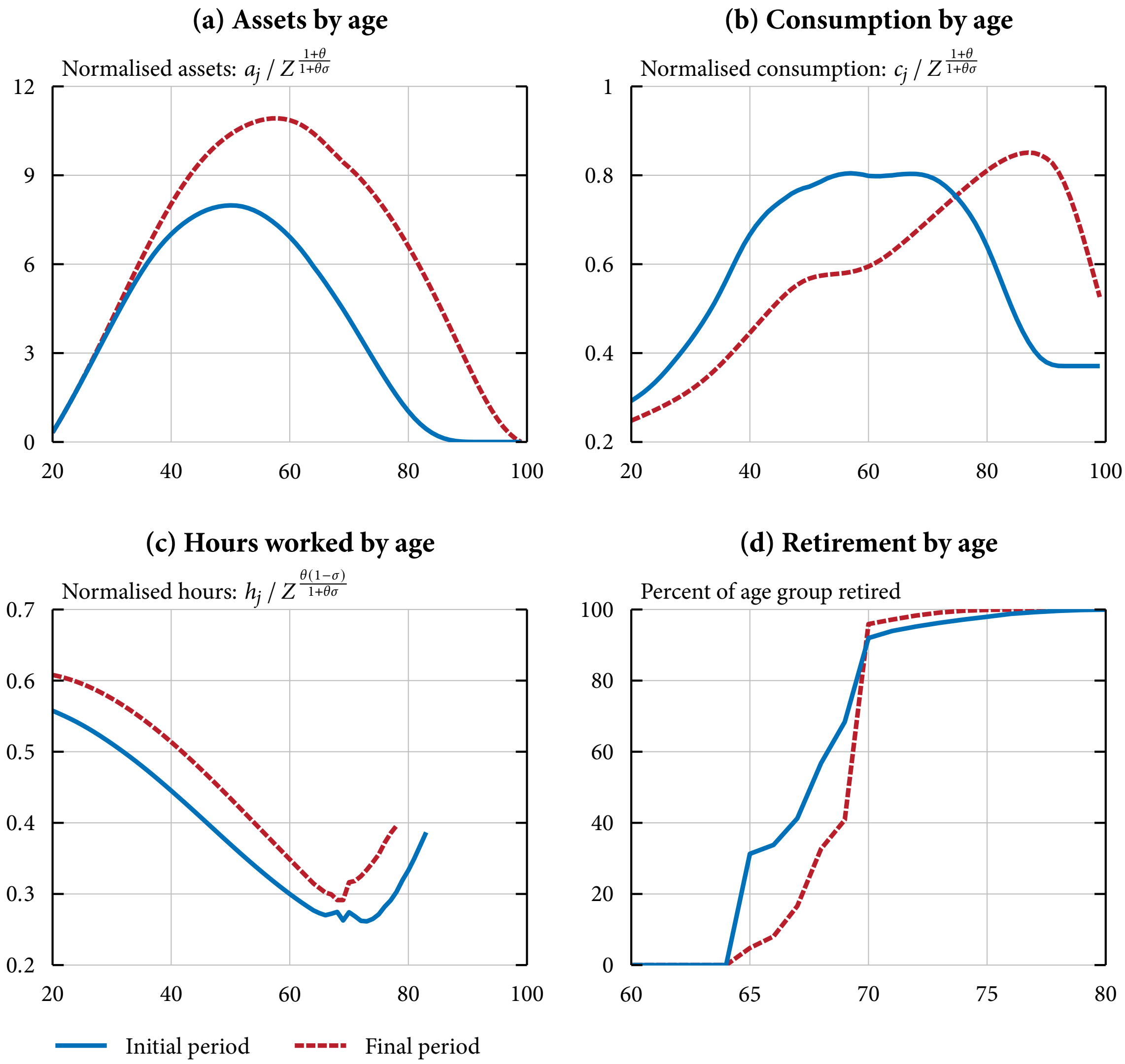

Figure E.3. Average life-cycle profiles in the baseline scenario.

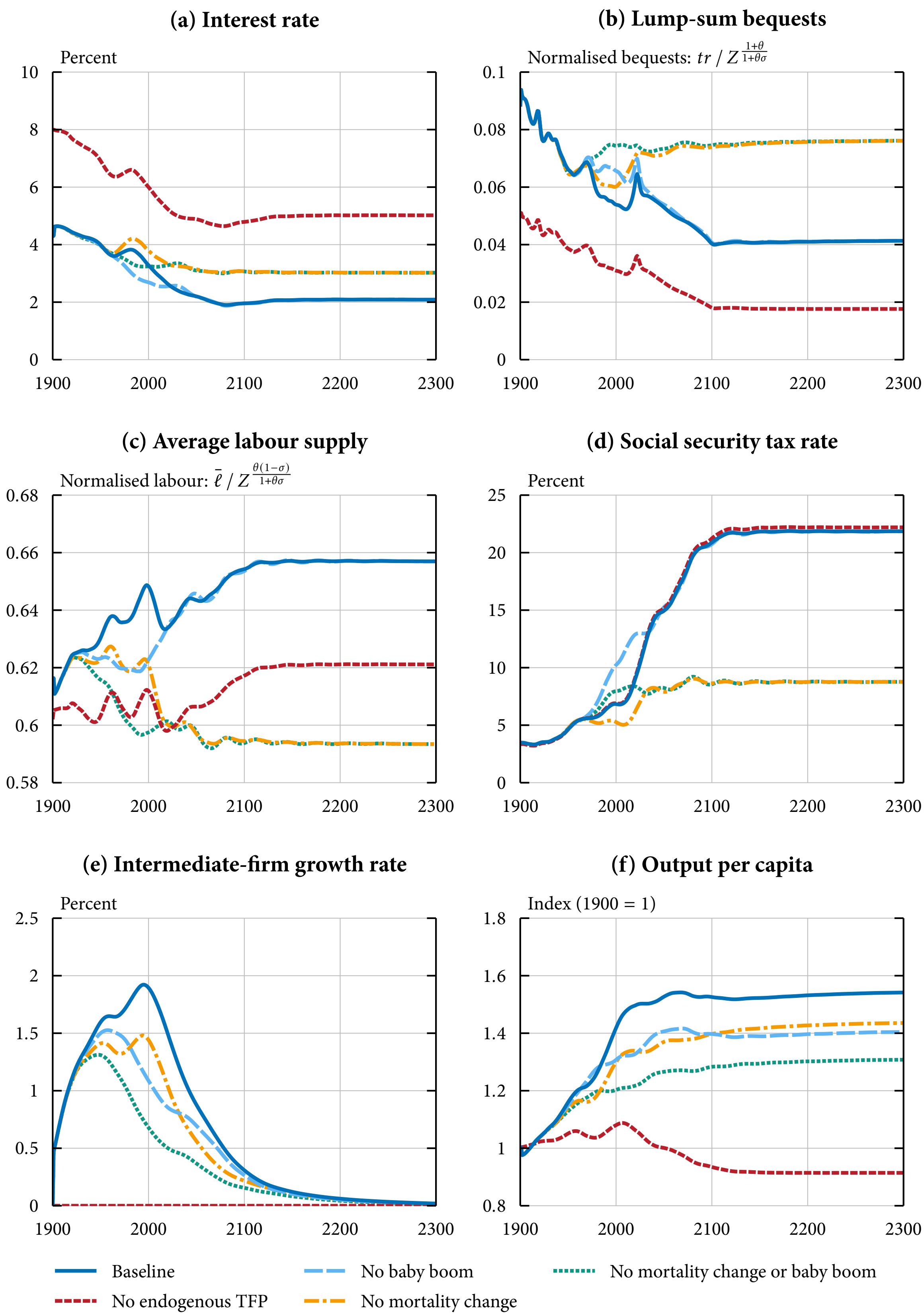

Figure E.4. Transition paths of equilibrium variables and output per capita.

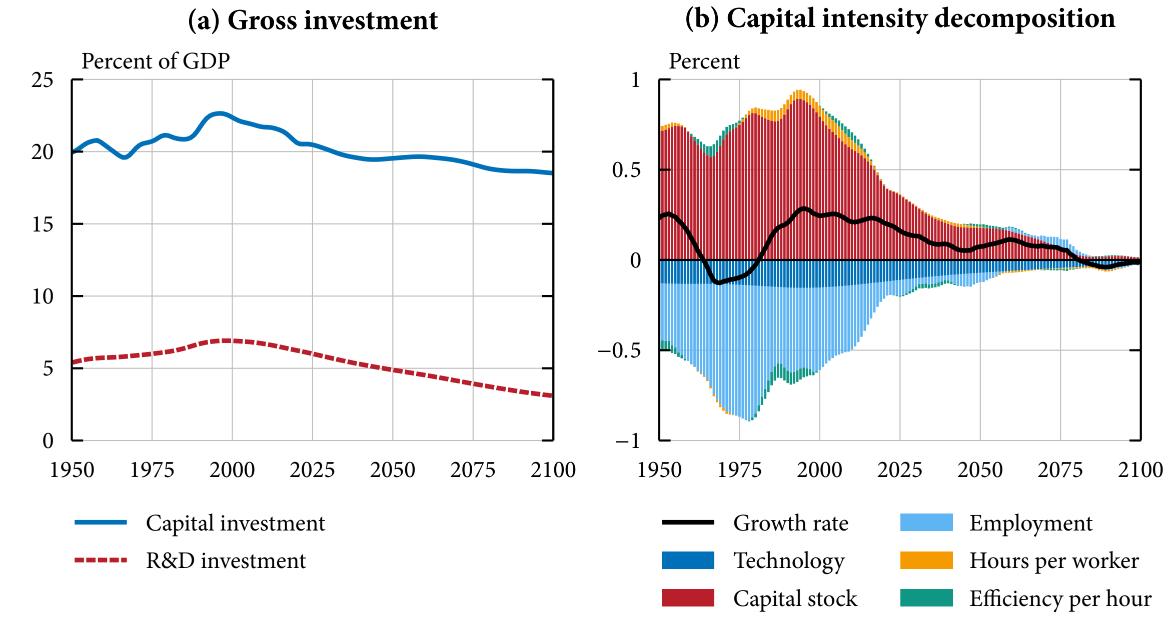

Figure E.5. Investment and capital deepening in the baseline scenario.

Notes. Figure E.5b displays the growth decomposition $\frac{\alpha}{1-\alpha} g_{K/Y} = \alpha(g_K - g_Z - g_E - g_h - g_\varepsilon)$, which is obtained by log differencing the capital intensity $\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{K}{ZL}\right)^{\alpha}$ together with the labour decomposition $L = E\,\bar{h}\,\bar{\varepsilon}$.

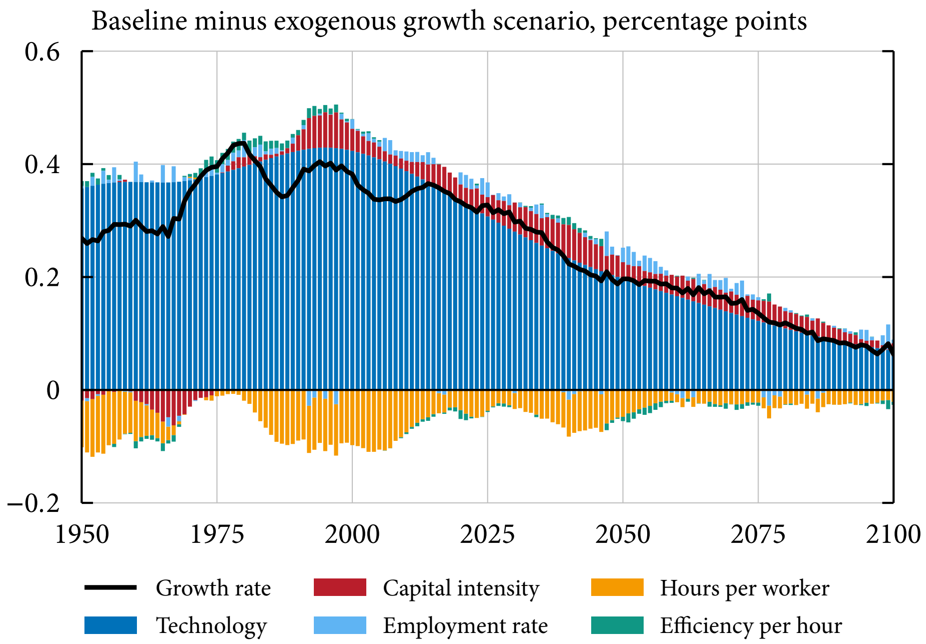

Figure E.6. Growth difference between the baseline and exogenous growth scenario.

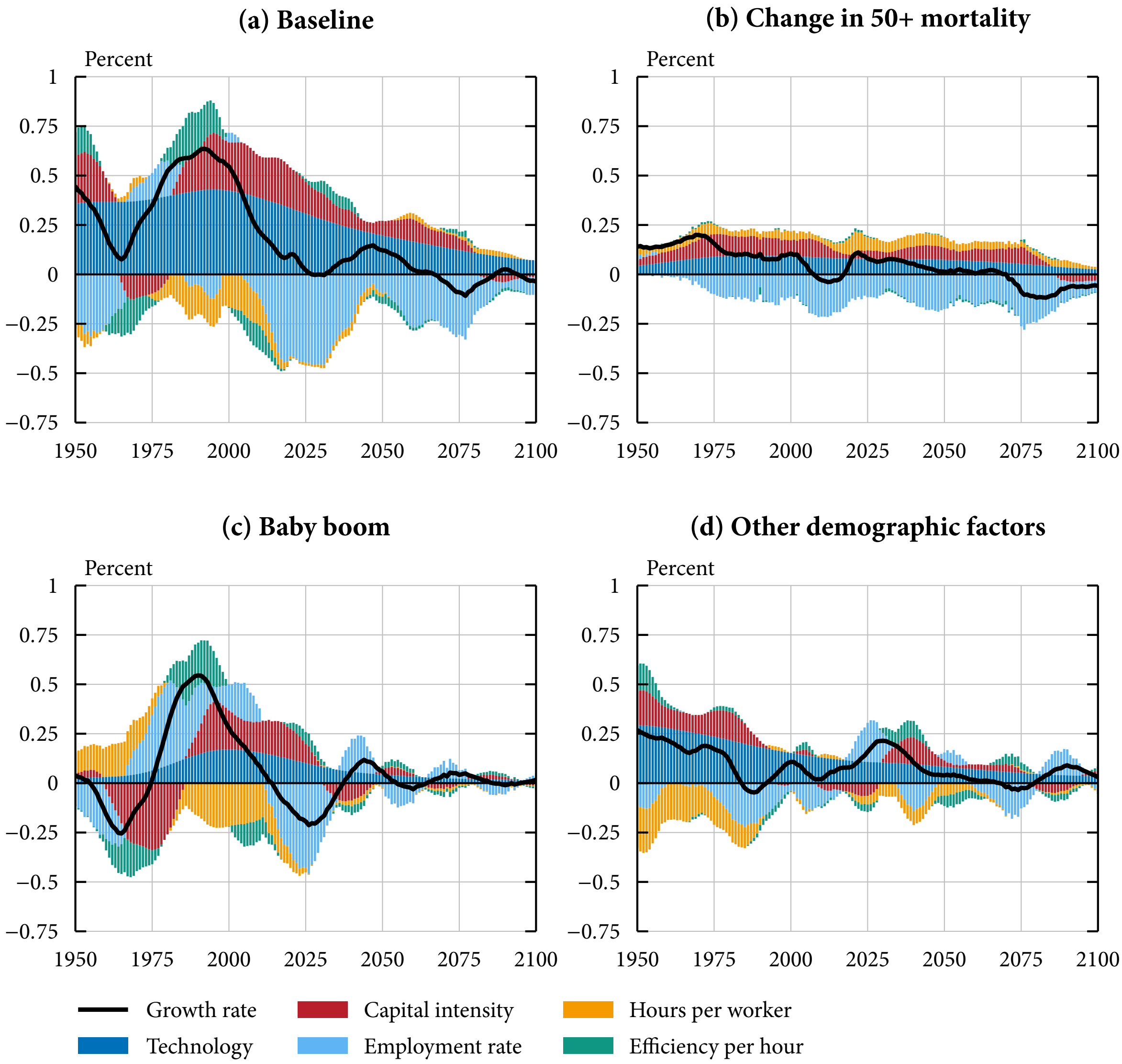

Figure E.7. Decomposing the growth impacts of the demographic forces.

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