Endogenous Technological Change Along the Demographic Transition*

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Abstract

Does population ageing hurt output per capita? Standard life-cycle models that consider two fundamental forces, capital deepening versus declining employment rates, predict yes. Using a quantitative overlapping generations model with R&D-driven growth, this paper challenges that prediction through a third possibility: that ageing populations boost R&D investment and therefore generate technological change. Calibrated to the United States, the model predicts that the demographic transition between 1950 and 2100 increases annual per-capita growth by 0.33 percentage points until 2000 and by 0.16 percentage points overall. The key mechanism is the endogenous technological change, whose growth contribution triples that of capital deepening.

Keywords: demographic transition, endogenous growth, OLG model.

JEL Codes: E17, E25, J11, O30, O40.

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1 Introduction

In 1950 there were 14 people aged 65 and above for every 100 people aged 20 to 64 in the United States. This number has risen steadily since and is projected to quadruple by the end of the twenty-first century in response to rising life expectancy and low fertility rates. Similar trends apply to high-income countries worldwide, with the rest of the world trailing not far behind. This dramatic shift raises serious macroeconomic concerns, and an important aspect underlying many of them is the impact of demographic change on national income.

How, then, does population ageing affect output per capita? Conventional wisdom based on life-cycle reasoning suggests two opposing channels. First, it reduces the fraction of workers in the population, thereby lowering output per capita. Second, rising life expectancy increases saving rates in anticipation of longer retirement, which spurs investment. The resulting capital deepening increases output per capita. Setting aside the severe identification issues between growth and demographics—let alone their underlying mechanisms—it therefore comes as no surprise that the empirical evidence on the net effect remains inconclusive.[1] Quantitative theory offers a solution to this ambiguity, and life-cycle models calibrated to advanced economies typically predict the first channel to dominate. Krueger and Ludwig (2007) and Ludwig, Schelkle and Vogel (2012) for instance find that demographic change during the twenty-first century generates cumulative declines in output per capita on the order of 10 percent.

This paper emphasises a third key channel that reverses both the sign and magnitude of the conventional prediction: the increase in savings may also finance investment in research and development (R&D), whose innovations improve output through technological progress. Figure 1 points to such a relationship by plotting saving rates and investment rates in intellectual property products against adult life expectancy for a sample of OECD countries, conditional on relative prices and dependency ratios. The data suggest that a one-year increase in life expectancy corresponds to a 0.20 percentage point higher investment rate in intellectual property products.[2] The distinction between R&D and capital investment is conceptually important because technology, unlike capital, is nonrival: an innovation can be used simultaneously by any number of workers at no additional cost and therefore raises everyone's productivity. Capital investment, by contrast, improves productivity only insofar as it increases capital per effective worker. Technological change, therefore, presents a stronger mechanism than capital deepening.

To formalize this argument and assess its quantitative significance, I develop a general equilibrium model that combines two standard macroeconomic frameworks. On the supply side, the model features endogenous growth à la Romer (1990), with monopolistically competitive intermediate producers and an R&D sector whose innovations enhance productivity by expanding the variety of intermediate goods. The household side follows in the Auerbach and Kotlikoff (1987) tradition with a large number of overlapping generations and a realistic population structure. These life-cycle households consume, save, and make labour supply decisions—both hours worked and the age of retirement—subject to changes in household size, mortality and income risk, borrowing constraints, progressive income taxation, and a bequest motive. Savings are either invested in physical capital or provide funding for R&D in exchange for ownership stakes in the new firms it generates, the latter creating the link between household behaviour and technological progress.

[1] See for instance Maestas, Mullen and Powell (2023) and the variety of estimates from the debate surrounding Acemoglu and Johnson (2007) by Lorentzen, McMillan and Wacziarg (2008), Aghion, Howitt and Murtin (2011), Cervellati and Sunde (2011), Bloom, Canning and Fink (2014), and Hansen and Lønstrup (2015).

[2] Using a panel of OECD countries from 1960 to 2011, Gehringer and Prettner (2017) also find that a 10 percent decrease in the crude death rate leads to a 1 percent increase in total factor productivity.

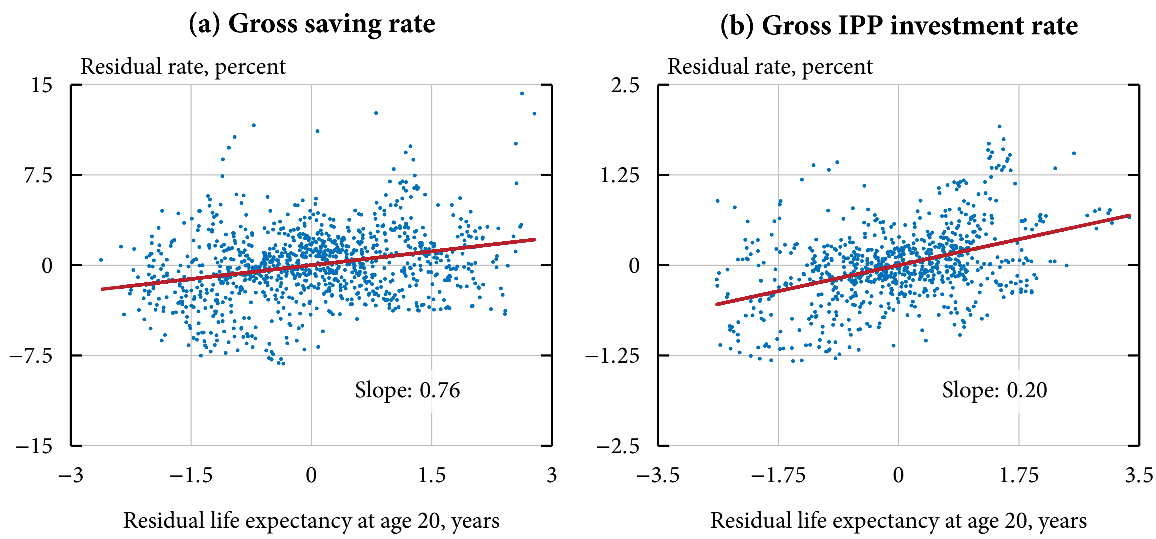


Figure 1. Partial correlations of savings, investment, and life expectancy, 1970–2020.

Notes. Data from the OECD and United Nations (2024) for countries that joined the OECD prior to 1975. IPP refers to intellectual property products (a superset of formal R&D). Each point represents a country-year observation after partialling out country fixed effects, relative prices, and dependency rates (0–14 and 65+ population shares). The red line shows the OLS fit.

The model exhibits semi-endogenous growth as in Jones (1995), Kortum (1997), Segerstrom (1998), and Peters and Walsh (2021), but its rich life-cycle structure departs from the representative-agent and perpetual-youth settings typically found in this literature. This contribution is central to the question at hand. In semi-endogenous growth theory, long-run economic growth is tied to population growth, which makes demographic analyses trivial as long as current trends truly reflect shifts in steady-state population growth. However, since World War II, demographic change in the advanced economies of North America and Western Europe primarily reflects two temporary forces: an initial surge in fertility (the baby boom) and steadily rising survival rates beyond age 50—well past childbearing years.[3] Neither affects long-run population growth when lifespans remain finite. Instead, these changes generate temporary growth effects that permanently affects the *level* of income, but such level effects cannot be properly examined through the lens of steady-state growth in a representative-agent or perpetual-youth model.

To shed light on these growth effects and to complement pure long-run analyses, this paper examines demographic changes around a fixed long-run trend, which ensures that key population trends are appropriately treated as purely transitory.[4] After illustrating the mechanisms in an analytically tractable three-generation model, I develop and calibrate the full quantitative model and simulate the equilibrium path induced by the demographic transition between 1950 and 2100. The analysis focuses on the United States—a large economy at the technological frontier—where the endogenous growth framework arguably provides a better description of technological change than in smaller economies that primarily adopt foreign technology. The United States also serves as a useful demographic benchmark, as its ageing trajectory lies between that of the fastest-ageing advanced economies and younger non-OECD countries.

Contrary to what a standard long-run analysis suggests, I find that the demographic transition is a net positive. On average, output per capita growth increases by 0.33 percentage points per year between

[3] Figure F.1 and Figure F.2 in Online Appendix F document these two facts. Besides the baby boom, fertility remains stable from the mid-1930s onward. Likewise, mortality during the first 50 years of life is largely unchanged after 1950.

[4] In doing so, I abstract from the possibility studied by Jones (2022a) that persistently low fertility could lead to a declining population—an important but ultimately separate issue.

1950 and 2000 and by 0.16 percentage points per year overall. The former corresponds to roughly 15–20 percent of observed US growth over the same period, thereby making demographics as important to US post-war growth as rising educational attainment according to estimates by Fernald and Jones (2014). Counterfactual simulations also reveal that the baby boom and rising life expectancy after the age of 50 explain around half of these results. Changes in the age structure due to the nonstationary population already present in 1950 and general increases in population size explain the remainder. As far as the impact on output per capita is concerned, these findings leave little reason to worry about current demographic change in general and about population ageing in particular.

The positive effect dissipates after the turn of the millennium when the baby boom retires, but the growth impact does not turn negative on average during the twenty-first century. Yet, the shift when this happens is sizeable: the growth rate declines by 0.53 percentage points between 1995 and 2030. Population ageing therefore exerts significant downward pressure on contemporary growth rates, just as Eggertsson, Lancastre and Summers (2019) and Jones (2023) argue in the context of secular stagnation. It is important to stress though that this decline is *not* a result of demographic change being inherently detrimental to economic output, but rather because of a reversion from the above-average growth that it induced in the first place. This interpretation stands in stark contrast to more pessimistic views.

At the core of these results lies technological change, which contributes three to four times more to output growth than capital deepening. This difference reflects the extra bang for the buck obtained from technology being nonrival. Together these two channels more than offset the negative impact of a growing share of retired households. Capital deepening alone cannot accomplish this: treating technology as exogenous (which is readily achieved as a special case of the benchmark model) eliminates the positive effect and leads to a 12.2 percent cumulative decline in output per capita between 1950 and 2100. This decline is quantitatively similar to Krueger and Ludwig (2007) and contrasts with a 27.5 percent increase in the baseline scenario, thus underlining that whether the demographic transition raises or lowers output per capita depends crucially on whether or not we account for its impact on technological progress.

The idea that the population age structure and longevity affect R&D-based growth is not new, dating back to at least Malmberg (1994). Among others, it is further emphasised by Prettner (2013), Gehringer and Prettner (2017), Prettner and Trimborn (2017), Baldanzi, Prettner and Tscheuschner (2019), Aksoy *et al.* (2019), and Basso and Jimeno (2021).[5] Like Section 2 of this paper, these studies rely on analytically tractable models, typically departing from perpetual-youth frameworks where retirement (if included at all) and mortality are independent of age. While useful for examining mechanisms, such models generate too stark household responses and lack the flexibility to match empirical age distributions, as explained by Carvalho, Ferrero and Nechio (2016) and Gagnon, Johannsen and López-Salido (2021). This limits their suitability for quantitative analyses compared to the full-fledged OLG model used here. Moreover, these studies generally interpret population ageing as a decline in steady-state population growth. As argued above, this distinction matters. For instance, Aksoy *et al.* and Basso and Jimeno find quantitatively similar growth declines to here during comparable decades and consequently conclude that population ageing reduces growth—the opposite of this paper's findings.

[5] Cutler *et al.* (1990) also suggest that ageing generates technological change via automation, which Acemoglu and Restrepo (2022) reiterate with evidence on two outcomes: robot adoption and the share of patents related to automation. Neither outcome contradicts the mechanisms here—the former reflects capital deepening, while the latter concerns the direction, not the volume, of innovation.

The focus on R&D-based growth within the context of demographic change also sets this paper apart from the broader quantitative macroeconomic literature, where this dimension is largely confined to business cycle analyses of a representative household.[6] By contrast, the extensive body of research that employs OLG models to study the macroeconomic effects of demographic change mainly concentrates on fiscal policy, international capital flows, and asset returns.[7] Studies like Krueger and Ludwig (2007) and Cooley and Henriksen (2018) that nevertheless touch upon the issue of growth typically restrict their attention to the trade-off between capital deepening and a shrinking workforce. While some papers incorporate additional adjustment margins, such as human capital accumulation (Ludwig, Schelkle and Vogel, 2012; Vandenbroucke, 2021) or automation (Heer and Irmen, 2014; Benzell *et al.*, 2021), no research to my knowledge conducts these quantitative analyses with endogenous R&D-driven technological change. This paper marks an initial step in this direction.

2 A Simple Model to Understand the Mechanisms

To build intuition, it is useful to first analyse a simplified version of the quantitative model that can be solved analytically. Thus, consider the Solow-esque world given in Table 1, in which households exogenously save a fixed fraction sr of income and supply labour L_t, which grows by the rate of the population n. Savings are allocated between next-period capital K_{t+1} and R&D investment Q_t. Together with the existing stock of technologies Z_t, R&D investment generates new technologies through an R&D production function as in Jones (1995). Output Y_t is determined by Cobb-Douglas production with capital, technology, and labour as inputs. For simplicity, both capital and technology fully depreciate each period, and a fixed fraction $\bar{\rho}$ of savings is invested in capital. With full depreciation of capital and technology, this fixed allocation of savings is also the market equilibrium if the model is expanded along the lines of Romer (1990) and Jones (1995) to include the returns on capital and R&D investments, but imposing it directly simplifies the exposition.

We are interested in how the demographic structure affects output per capita $y_t \equiv Y_t/N_t$, where N_t is the size of the population. A helpful decomposition of per-capita output based on the Cobb-Douglas production function is then

$$y_t = Z_t \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}, \tag{1}$$

which shows that output per capita is proportional to the stock of technology (total factor productivity, TFP), a capital intensity (captured by the capital-output ratio), and the employment rate. Recalling that the demographic transition is characterised by two salient features after 1950—the baby boom and increasing life expectancy after age 50—the focus of the following subsections is to understand hos these demographic trends affect the components of Equation (1).

2.1 Components of Output Per Capita in Steady State

Table 1 outlines a semi-endogenous growth model in which the components of Equation (1) can be solved explicitly in steady state—an equilibrium where all variables grow at constant (possibly zero) rates. First, dividing (S2) by Z_t yields the gross TFP growth rate $1+g_{Zt} \equiv \frac{Z_{t+1}}{Z_t} = Q_t^{\lambda} Z_t^{\phi-1}$. In steady state, the right-hand

[6] Examples include Comin and Gertler (2006), Nuño (2011), Benigno and Fornaro (2018), Anzoategui *et al.* (2019), Bianchi, Kung and Morales (2019), and Okada (2022).

[7] Besides papers mentioned elsewhere, a far from exhaustive list includes De Nardi, İmrohoroğlu and Sargent (1999), Storesletten (2000), Fehr, Jokisch and Kotlikoff (2004), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006b), Attanasio, Kitao and Violante (2007), Kotlikoff, Smetters and Walliser (2007), İmrohoroğlu and Kitao (2012), Kitao (2014), Gagnon, Johannsen and López-Salido (2021), and Auclert *et al.* (2024).

Table 1. Summary of the simple model.

Output	$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}$,	$0 < \alpha < 1$	(S1)
R&D output	$Z_{t+1} = Q_t^{\lambda} Z_t^{\phi}$,	$0 < \lambda \leq 1,\ \phi < 1$	(S2)
Asset market	$sr\, Y_t = K_{t+1} + Q_t$		(S3)
Allocation	$Q_t = (1-\bar{\rho})\, sr\, Y_t$,	$0 < \bar{\rho} < 1$	(S4)
Population growth	$L_{t+1} = (1+n)\, L_t$		(S5)

side must remain constant while R&D investment must grow at the rate of total output. It follows that steady-state TFP growth satisfies

$$1 + g_Z = (1+n)^{\gamma}, \qquad \text{where} \qquad \gamma \equiv \frac{\lambda}{1-\phi-\lambda}, \tag{2}$$

and this is a function of exogenous parameters, hence the "semi-endogenous" modifier. Combining Equation (2) with (S3) and (S4) gives the capital-output ratio:

$$\frac{K_t}{Y_t} = \frac{\bar{\rho}\, sr}{(1+n)^{\frac{\gamma(1-\phi)}{\lambda}}}. \tag{3}$$

Like the Solow model, the capital intensity is thus the share of output allocated to capital investment divided by the aggregate growth rate. Equations (2) and (3) can then be used with those in Table 1 to solve for the stock of TFP:

$$Z_t = \left[\frac{1-\bar{\rho}}{\bar{\rho}} \left(\frac{\bar{\rho}\, sr}{(1+n)^{\frac{\gamma(1-\alpha\phi)}{\lambda}}} \right)^{\frac{1}{1-\alpha}} \right]^{\gamma} L_t^{\gamma}. \tag{4}$$

Equation (4) writes TFP as the product of two factors. The first is an R&D intensity, which is determined by the saving rate and thus reflects household behaviour and composition. The second, L_t^{γ}, is the scale effect on output per capita present in all semi-endogenous growth models (Jones, 2005). Specifically, for given household behaviour, a larger population increases aggregate investment, and this raises the stock of TFP. But technology is nonrival; once discovered, it can be used simultaneously by everyone at no additional cost. Output per capita therefore depends on total TFP rather than TFP per capita, as seen in Equation (1). Consequently, output per capita increases simply by virtue of a larger population.

At this point, let's add households to endogenise the employment and saving rates. Consider a life-cycle model with young, middled-aged, and old households (indexed by $j = 1, 2, 3$, with cohort size N_{jt}). Young and middle-aged households supply labour inelastically, while old households are retired. If the size of the young cohort grows at rate $1 + n$ each period, and a j-year-old survives to age $j + 1$ with probability s_j, then the steady-state employment rate is

$$\frac{L_t}{N_t} = \frac{N_{1t} + N_{2t}}{N_{1t} + N_{2t} + N_{3t}} = \frac{1 + \frac{s_1}{1+n}}{1 + \frac{s_1}{1+n}\left(1 + \frac{s_2}{1+n}\right)}. \tag{5}$$

For simplicity, suppose households have logarithmic preferences over consumption c, that annuity markets are present, and that young households consume their entire income. Households then maximise expected lifetime utility $\log(c_1) + s_1\beta\log(c_2) + s_1s_2\beta^2\log(c_3)$ subject to the budget constraints $c_1 = w$, $c_2 + a = w$, and $c_3 = \frac{1+r}{s_2}a$, where β, a, r, and w denote the discount factor, household assets, the interest rate, and the wage rate, respectively. The solution to the household problem is characterised by the savings policy $a_{t+1} = \frac{s_2\beta}{1+s_2\beta}w_t$. The aggregate steady-state saving rate can then be written

$$sr = \frac{a_{t+1}N_{2t}}{Y_t} = \frac{w_tL_t}{Y_t}\cdot\frac{s_3\beta}{1+s_3\beta}\cdot\frac{\frac{s_2}{1+n}}{1+\frac{s_2}{1+n}}, \tag{6}$$

which is a product of the labour share of national income, the share of labour income allocated to savings, and the share of middle-aged workers (i.e., savers) in the labour force. Note that the labour share of income is constant given the Cobb-Douglas production function with constant exponents.

2.2 Population Ageing via Increasing Survival Rates

Consider now an ageing of the population through increases in the survival rates s_1 and s_2. Think of the former as representing the increase in survival rates for ages 50–64 observed in the data, while the latter represents the similar increase observed for ages 65 and above. The higher survival rates naturally push down the employment rate through a mechanical increase in the old-age dependency ratio, as seen in Equation (5). This reduces output per capita by (1). Meanwhile, Equations (3), (4) and (6) highlight a counteracting force: higher survival rates raises the saving rate, which in turn generates larger capital and R&D intensities, the latter of which improves TFP. Additionally, an increase in the middle-age survival rate s_1 also raises the size of the labour force relative to its previous trend, thereby increasing TFP further through the scale effect present in (4).

The intuition for the effects on the capital and R&D intensities is straightforward. A higher likelihood of middle-age survival raises the share of households that save, while a higher likelihood of old-age survival increases the life-cycle savings motive, so households save more intensively. The extra savings are allocated proportionately to capital and R&D investment according to (S4). The capital stock, however, increases disproportionately more than TFP because the production of capital is linear in investments whereas the R&D process features decreasing returns to scale: a necessary parameter restriction for a steady state to exist is $\gamma > 0$, which holds for feasible values of λ and ϕ if and only if $\lambda + \phi < 1$. Therefore, both K_t/Y_t and Z_t rise.

2.3 A Baby Boom via Temporarily Increasing Fertility Rates

Next consider a temporary increase in the population growth rate $1 + n$, similar to a baby boom. Since population growth eventually returns to its original level, this leaves the long-run age structure unchanged. As a result, neither the employment rate nor the saving rate (and, by extension, the capital and R&D intensities) is affected. The only long-run impact is through the scale effect in Equation (4): a temporary fertility boom permanently raises the population size, which leads to higher TFP and subsequently higher output per capita. This is exactly what a standard semi-endogenous growth model with a representative household predicts.

Equations (3) to (6) nevertheless provide guidance about the transition dynamics beyond the scale effect for this scenario. The saving rate is low when the share of middle-aged households is low, and the employment rate is high when the share of old households is low. The initial boost in the share of young households therefore generates positive growth via the employment rate and reduced growth via the capital and R&D

Table 2. Comparative statics of the simple model.

Change	Duration	Output per capita	Technology: R&D intensity	Technology: Scale	Capital intensity	Employment rate
Mid-age mortality: $s_1 \uparrow$	Permanent	Ambiguous	+	+	+	−
Old-age mortality: $s_2 \uparrow$	Permanent	Ambiguous	+		+	−
Fertility: $1 + n \uparrow$	Temporary	+		+		

Notes. Increases denoted by plus signs, decreases denoted by minus signs.

intensities. As this generation reaches middle age and their children enter the labour force, the positive impact on the employment rate remains at the same time as the saving rate recovers, thus reverting the effect on the capital and R&D intensities. Finally, when this generation reaches retirement, the employment rate declines back to its initial level, which completes the transition. These predictions will be useful to understand the quantitative results in later sections.

2.4 Taking Stock

The steady-state predictions are summarised in Table 2. Overall, the demographic transition lowers the employment rate through the old-age dependency ratio, raises the capital intensity via an increased saving rate, and generates technological change through both the saving rate and the scale effect associated with the nonrivalry of technology. Yet, the relative strengths of these channels and the net effect on transitory growth are ambiguous. This motivates the need for a quantitative treatment.

3 A Quantitative OLG Model With Endogenous Growth

The quantitative model is a closed economy populated by overlapping generations of households, production firms, R&D firms, and a government. Time is discrete and a period is one year. Exogenous change in the demographic structure constitutes the driving force of the model.[8] The model features a rich set of factors that affect household behaviour over the life cycle, including changes in household size, mortality risk, idiosyncratic income risk, borrowing constraints, progressive income taxation, intensive (hours worked) and extensive (age of retirement) margins of labour supply, and bequest motives. The rest of the model remains close to the growth literature: the production sector consists of a perfectly competitive final-good firm and monopolistic intermediate-good firms, and technology grows endogenously through the entry of new intermediate firms created from innovations in an R&D sector.[9]

3.1 Demographics and Household Composition

In period t, the economy consists of $J + 1$ generations of sizes $N_{0t}, \ldots, N_{Jt}$, and the total population is $N_t = \sum_{j=0}^{J} N_{jt}$. From a given initial population, the demographic structure in subsequent periods is pinned

[8] For simplicity, I abstract from the possibility that technological change reversely affects demographic variables (birth control, treatment of diseases, and so on). This follows Lee (2016, p. 111), who states "My personal view is that trajectories of fertility, mortality, and health should be taken as exogenous. While theories are available to relate these to individual choices, they have little predictive power and their use might obscure the workings of some better understood mechanisms."

[9] The mechanisms emphasised in this paper are present in any standard endogenous growth model, so choosing between the expanding-variety model of Romer (1990) or a quality ladder à la Aghion and Howitt (1992) and Grossman and Helpman (1991) is inconsequential here.

down recursively by age and time specific fertility rates f_{jt}, survival rates s_{jt}, and net migration rates m_{jt} according to

$$N_{0,t+1} = \sum_{j=0}^{J} f_{jt} N_{jt} \qquad \text{and} \qquad N_{j+1,t+1} = \big(s_{jt} + m_{jt}\big) N_{jt}, \tag{7}$$

where $s_{Jt} = m_{Jt} = 0$ holds for all t, implying that individuals die with certainty after age J. Migrants bring their accumulated wealth when they move and are economically identical to non-migrants. This assumption eliminates the need to separate between natives and migrants in the economic model.

Individuals are children for the first ι years of their lives, after which they form a household and become economically active. A household consists of an adult head and their dependants, including both children and grandchildren of underaged parents. For an adult of age j, the number of dependants of age $i < \iota$ is given by

$$N_{ijt}^{d} = f_{j-1-i,t-1-i} \frac{N_{j-1-i,t-1-i}}{N_{0,t-i}} \frac{N_{it}}{N_{jt}} + \sum_{k=i+1}^{\iota-1} N_{kjt}^{d} N_{ikt}^{d}. \tag{8}$$

The first term is the number of children. It is obtained by taking the fraction of the i-year-olds born to parents who are now age j, multiplying it by the current population of i-year-olds, and distributing that total evenly across the current j-year-old population. The second term is the number of grandchildren, where the number of i-year-old dependants of an underaged parent is given analogously to the first term on the right-hand side of (8).

3.2 Household Endowments and Preferences

Individuals are endowed with one unit of time and enter adulthood as members of the labour force with no assets. Workers provide an effective labour supply of $\ell_j = \varepsilon_j \eta h_j$ efficiency units, where h_j is hours worked, ε_j is a deterministic, age-dependent productivity level, and η is an idiosyncratic productivity component that evolves stochastically following a time-invariant Markov chain. Idiosyncratic productivity in the initial adult age ι is distributed according to the invariant distribution of the associated Markov chain, and households can partially self-insure against idiosyncratic shocks through asset holdings a subject to a borrowing constraint. Additionally, there are no annuity markets to insure against mortality risk. Instead, the government seizes the assets of households that die prematurely and redistributes them as lump-sum transfers tr to surviving households. Leaving time subscripts implicit in this subsection, the budget constraint facing a household head at age j is thus written

$$a_{j+1} + (1+\tau^c)c_j = \big(1 + r(1-\tau^k)\big)a_j + \big(1 - \tau^w(w\ell_j) - \tau^b\big)w\ell_j + tr + b(R_j), \tag{9}$$

where c_j is total household consumption, r is the rate of return on savings, w is the wage rate, and τ^c, τ^k, τ^w, τ^b denote tax rates where, in particular, τ^w is allowed to vary with labour income. The final term, $b(R_j)$, is a pension benefit to be specified later that depends on whether a person is retired and on the age of retirement, both of which are captured by the retirement status R_j.

A household head of age j values their own consumption c_j^a, their leisure time $1 - h_j$, the consumption c_{ij}^d per i-year-old dependant, and the warm glow of knowing a bequest is left in case of future death. Starting with consumption, its flow utility is generated by

$$u_c\left(c_j^a,\ \{c_{ij}^d\}_{i=0}^{\iota-1}\right) = \frac{\left(c_j^a\right)^{1-\sigma}-1}{1-\sigma} + \sum_{i=0}^{\iota-1}\omega_i N_{ij}^d \frac{\left(c_{ij}^d\right)^{1-\sigma}-1}{1-\sigma}, \tag{10}$$

where $1/\sigma$ is the intertemporal elasticity of substitution for consumption and ω_i is the utility weight of i-year-old dependants. Households benefit from economies of scale in the sense that total household consumption satisfies the constant elasticity of transformation function

$$c_j = \left[\left(c_j^a\right)^\zeta + \sum_{i=0}^{\iota-1} N_{ij}^d\left(c_{ij}^d\right)^\zeta\right]^{\frac{1}{\zeta}}, \qquad \zeta \geq 1, \tag{11}$$

in which the parameter ζ captures the degree of scale. If $\zeta = 1$, there are no scale benefits and total household consumption is just the sum of the individual consumption levels. Equations (10) and (11) admit an equivalent utility function expressed in terms of total household consumption. Taking household consumption as given and maximising (10) subject to (11) with respect to individual allocations, we obtain $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$. This condition combined with (10) and (11) generates a utility function of the form

$$u_c(c_j) = \Omega_{jc}\frac{c_j^{1-\sigma}-1}{1-\sigma}, \qquad \text{where} \qquad \Omega_{jc} \equiv \left[1 + \sum_{i=0}^{\iota-1}\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}} N_{ij}^d\right]^{\frac{\zeta-(1-\sigma)}{\zeta}} \tag{12}$$

is an age and time specific household size taste shifter. Specifically, substituting $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$ into Equation (11) reveals that $c_j = \Omega_{jc}^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$, so $\Omega_{jc}^{\frac{1}{\zeta-(1-\sigma)}}$ measures the number of adult equivalents in a household and $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ is the equivalence weight of an i-year-old dependant. In terms of household consumption, leisure, and bequests, overall utility is subsequently defined as

$$u(c_j, h_j, a_{j+1}^b) = \Omega_{jc}\frac{c_j^{1-\sigma}-1}{1-\sigma} - \psi\frac{h_j^{1+1/\theta}}{1+1/\theta} + (1-s_j)\,\Omega_a\frac{\left(a_{j+1}^b\right)^{1-\sigma}-1}{1-\sigma}, \tag{13}$$

where θ is the Frisch elasticity of labour supply, ψ determines the disutility of working, Ω_a is the utility weight of bequests, and $a_{j+1}^b \equiv \left(1 + r(1-\tau^k)\right)a_{j+1}$ are next-period bequests. Equation (13) belongs to the class of balanced growth preferences characterised by Boppart and Krusell (2020) that allows for falling hours worked in the long run; the income effect of higher wages on leisure dominates the substitution effect if $\sigma > 1$, leading to a decline in hours worked when long-run wage growth is positive. These preferences are summed into an expected lifetime utility function of the form

$$\mathbb{E}\left[\sum_{j=\iota}^{J}\beta^{j-\iota}\left(\prod_{k=\iota}^{j-1}s_k\right)u\left(c_j, h_j, a_{j+1}^b\right)\right], \tag{14}$$

where β is the discount factor and expectations are taken over idiosyncratic productivity.

Households maximise expected lifetime utility (14) by choosing consumption, hours worked, and their age of retirement. At the start of each period, households observe their state $x'_j = (a_j, \eta, R_{j-1})$ consisting of current wealth and idiosyncratic productivity and the previous-period retirement status, and subsequently make their retirement decisions. Consumption and hours worked are then chosen in a second stage conditional on the post-retirement decision state $x_j = (a_j, \eta, R_j)$. Retirement is an absorbing state, so the labour supply choices during retirement are trivial and the household problem reduces to a standard consumption-savings choice.

Formally, denote the retirement choice by a discrete variable d equal to 1 if choosing to remain in the labour force and 0 otherwise and let the retirement status evolve by $R_j = R_{j-1} + d_j$, starting with $R_{\iota-1} = \iota$. An optimal retirement policy is then a function $d_j(x'_j)$ that solves the first-stage problem

$$V_j(x'_j) = \max_{d_j \in D(R_{j-1})} \left\{ v_j(x_j) \right\} \tag{15}$$

subject to the motion of the retirement status and to

$$D(R_{j-1}) = \begin{cases} \{0, 1\} & \text{if } R_{j-1} = j, \\ \{0\} & \text{if } R_{j-1} < j, \end{cases}$$

where $V_j(x'_j)$ and $v_j(x_j)$ are the pre and post retirement decision value functions at age j. Optimal policies for consumption, savings, and hours worked are functions $c_j(x_j)$, $a_j(x_j)$, and $h_j(x_j)$ that solve the second-stage problem

$$v_j(x_j) = \max_{c_j, h_j} \left\{ u\left(c_j, h_j, a^b_{j+1}\right) + s_j \beta\, \mathbb{E}\left[V_{j+1}(x'_{j+1}) \,\middle|\, \eta \right] \right\} \tag{16}$$

subject to the budget constraint (9), the time constraints $h_j \in [0, 1]$ if working and $h_j = 0$ if retired, and the borrowing constraint $a_{j+1} \geq 0$.

3.3 Production

The competitive final-good sector hires labour L_t at the wage w_t and buys a continuum of intermediate capital inputs $\{k_{it}\}$ indexed by i at prices $\{p_{it}\}$ to produce output Y_t. Production follows the Cobb-Douglas function

$$Y_t = L_t^{1-\alpha} \left(\int_0^{z_t} k_{it}^{\rho}\, di \right)^{\frac{\alpha}{\rho}}, \qquad 0 < \alpha, \rho < 1, \tag{17}$$

where z_t is a measure of the intermediate varieties available at time t. Intermediate-good firms have a linear production function that converts capital into intermediate inputs one for one. Capital is rented from households at the rate $r_t + \delta_k$, where δ_k is the capital depreciation rate. Each firm in the intermediate sector has a patent for their own variety and acts as a monopolist. Conditional on having a patent, a firm j consequently maximises operating profits $\pi_{jt} = (p_{jt} - r_t - \delta_k)k_{jt}$ subject to the final-good producer's demand for j. By profit maximisation in the final-good sector, this demand constraint is given by

$$p_{jt} = \alpha \frac{Y_t}{k_{jt}} \frac{k_{jt}^{\rho}}{\int_0^{z_t} k_{it}^{\rho}\, di}. \tag{18}$$

By symmetry, all firms charge the same mark-up over marginal cost, $p_{it} = p_t = \frac{1}{\rho}(r_t + \delta_k)$ and therefore sell the same quantity k_t and earn the same profits $\pi_t = (1-\rho)p_t k_t$.

3.4 The R&D Sector

The R&D sector exhibits free entry and a unit measure of innovators invest an amount Q_t of final output to develop new designs of intermediate goods. Patents for new designs are sold to prospective intermediate-good firms at the end of a period t for a one-off price $P_{z,t+1}$, which then convert the designs into usable input in period $t+1$. These patent purchases are financed by households in exchange for equity ownership in the intermediate-good firms. There is free entry into R&D and, like the simple model in Section 2, innovation is characterised by an overall production function as in Jones (1995):

$$F(Q_t) = \bar{v}_t Q_t = v Q_t^{\lambda} z_t^{\phi}, \qquad 0 < \lambda \leq 1, \ \phi < 1. \tag{19}$$

The productivity factor $\bar{v}_t \equiv v Q_t^{\lambda-1} z_t^{\phi}$ incorporates duplication effects (via λ) and knowledge spillovers (via ϕ) that impact the aggregate development but are external to individual innovators. R&D firms thus choose expenditures Q_t to maximise profits $(P_{z,t+1}\bar{v}_t - 1)Q_t$, which together with free entry yields the zero-profit condition

$$Q_t = P_{z,t+1} v Q_t^{\lambda} z_t^{\phi}. \tag{20}$$

As in Comin and Gertler (2006) and similar papers, an intermediate-good firm is not infinitely lived; each period, a fraction δ_z of firms become obsolete.[10] The law of motion of new intermediates is therefore $z_{t+1} = (1-\delta_z)z_t + F(Q_t)$, thus implying the gross growth rate of intermediate varieties

$$1 + g_{zt} = 1 - \delta_z + v Q_t^{\lambda} z_t^{\phi-1}. \tag{21}$$

Lastly, a prospective intermediate producer enters only if it is profitable to do so. That is, the firm enters if the sum of expected discounted flow profits π exceeds the fixed cost P_z of purchasing a patent. Free entry into the intermediate-good sector drives the profitability of entry to zero. This is equivalent to saying that the following no-arbitrage condition holds in equilibrium:

$$r_t = \frac{\pi_t + \Delta P_{zt} - \delta_z P_{z,t+1}}{P_{zt}}, \tag{22}$$

where $\Delta P_{zt} \equiv P_{z,t+1} - P_{zt}$ is the change in the patent price in period t.

3.5 The Public Sector

The public sector engages in three activities: it redistributes assets from deceased individuals; it collects taxes on consumption, capital gains, and wages via the tax rates τ_t^c, τ_t^k, and τ_t^w to finance public consumption G_t; and it operates a pay-as-you-go social security system, which is financed by contribution rates τ_t^b on labour earnings. The budget constraint of each public sector activity is independent of the other two and always balances. In the baseline model, budget balance is achieved through endogenous adjustments in the bequest transfers tr, public consumption G_t, and social security contribution rates τ_t^b. An ageing population in this setup generates higher contribution rates to maintain pension benefits. Compared to the alternative (constant tax rates and cutting benefits), this is the most growth pessimistic arrangement, since

[10] This feature is primarily technical to calibrate the level of R&D investment. We can also set $\delta_z < 0$ to generate exogenous growth, as in the standard neoclassical model, in addition to that created through R&D.

it negatively impacts both the incentive to save for retirement and for late retirement (due to generous pensions) as well as the ability to save (through higher taxation).

Pensions are independent of earnings history but depend on the age of retirement.[11] Specifically, a household with retirement status R receives a pension that is scaled by a factor $ps(R)$ relative to a base-level benefit received if retiring at some normal retirement age R^{norm}, thus allowing for early retirement penalties and delayed retirement credits. The base-level benefit is a fraction μ of average gross labour income $w_t\bar{\ell}_t$, where $\bar{\ell}_t$ is the average number of efficiency units per worker. An age-j household therefore receives a pension transfer

$$b_t(R) = \begin{cases} ps(R)\mu\, w_t\bar{\ell}_t & \text{if } j \geq \max\{R^{min}, R\}, \\ 0 & \text{if } j < \max\{R^{min}, R\}, \end{cases} \tag{23}$$

where no pension is paid out to retirees younger than some threshold age R^{min} or to workers.

3.6 Equilibrium

In any period t, the state of the economy is pinned down by the aggregate capital stock $K_t \equiv \int_0^{z_t} k_{it}\, di$, the number of intermediate varieties z_t, the patent price P_{zt}, and measures $\{\Phi_{jt}(x)\}_{j=\iota}^{J}$ characterising the distribution of households after their retirement decisions are made. Since the state of a household is determined by their asset wealth, idiosyncratic productivity, and retirement status, the state space for the household measures is given by $X = \mathbb{R}_+ \times H \times \{\iota, \ldots, J+1\}$, where H denotes the state space of the Markov chain for idiosyncratic productivity. Given a demographic evolution and initial conditions K_0, z_0, $P_{z,0}$, $\{\Phi_{j0}(x)\}_{j=\iota}^{J}$, we then have:

Definition 1 (General equilibrium)**.** An equilibrium consists of paths for household decision rules $\{d_{jt}(\cdot)$, $c_{jt}(\cdot)$, $a_{jt}(\cdot)$, $h_{jt}(\cdot)\}_{t=0}^{\infty}$ and measures $\{\Phi_{jt}(\cdot)\}_{t=0}^{\infty}$ for all ages j, prices and profits $\{r_t, w_t, \pi_t, P_{zt}\}_{t=0}^{\infty}$, taxes and transfers $\{\tau_t^c, \tau_t^k, \tau_t^b, \tau_t^w(\cdot), tr, b_t(\cdot)\}_{t=0}^{\infty}$, and aggregate quantities such that for all periods t:

(i) Household decision rules solve problems (15) and (16).

(ii) Profit maximisation of final and intermediate good firms yields the consolidated production function

$$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}, \qquad \text{where} \qquad Z_t \equiv z_t^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}, \tag{24}$$

with corresponding factor prices and profits

$$w_t = (1-\alpha)\frac{Y_t}{L_t}, \qquad r_t = \alpha\rho\frac{Y_t}{K_t} - \delta_k, \qquad \pi_t = \alpha(1-\rho)\frac{Y_t}{z_t}. \tag{25}$$

(iii) The measure of intermediate varieties z_t, the patent price P_{zt}, and aggregate R&D investment Q_t satisfy Equations (20) to (22).

(iv) The public sector balances its budgets:

$$G_t = \tau_t^c C_t + \tau_t^k r_t A_t + \sum_{j=\iota}^{J} N_{jt} \int_X \tau^w\big(w_t \ell_{jt}(x)\big) w_t \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{26}$$

[11] Earnings-dependent pension introduces another continuous state variable in the household problem and I abstract from this feature to avoid the additional computational complexity that it entails, as is common in Auerbach and Kotlikoff (1987).

$$\tau_t^b w_t L_t = \sum_{j=\iota}^{J} N_{jt} \int_X b_t(R)\, \mathrm{d}\Phi_{jt}, \tag{27}$$

$$tr_{t+1} = \frac{1 + r_{t+1}(1-\tau_{t+1}^k)}{\sum_{j=\iota}^{J} N_{j,t+1}} \sum_{j=\iota}^{J} (1-s_{jt}) N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{28}$$

where $C_t = \sum_{j=\iota}^{J} N_{jt} \int_X c_{jt}(x)\, \mathrm{d}\Phi_{jt}$ is aggregate consumption and $A_{t+1} = \sum_{j=\iota}^{J} (1+m_{jt}) N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt}$ is aggregate wealth.

(v) Labour, capital, and goods markets clear:

$$L_t = \sum_{j=\iota}^{J} N_{jt} \int_X \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{29}$$

$$A_t = K_t + P_{zt} z_t, \tag{30}$$

$$Y_t + A_{t+1}^M = C_t + G_t + \left[K_{t+1} - (1-\delta_k)K_t\right] + Q_t, \tag{31}$$

where $A_{t+1}^M = \sum_{j=\iota}^{J} m_{jt} N_{jt} \int_X a_{jt}(x) \mathrm{d}\Phi_{jt}$ is the net wealth brought by migrants.

(vi) For a given Markov kernel Π and for all Borel sets $S = \mathcal{A} \times \mathcal{H} \times \mathcal{R}$ on X (and dropping time subscripts), the household distributions evolve according to

$$\Phi_{j+1}(S) = \int_{X_W} \left[\int_{\mathcal{H}} d_{j+1}(x')\, \Pi(\eta, \mathrm{d}\eta') \right] \mathrm{d}\Phi_j + \int_{X_R} \left[\int_{\mathcal{H}} \left(1 - d_{j+1}(x')\right) \Pi(\eta, \mathrm{d}\eta') \right] \mathrm{d}\Phi_j, \tag{32}$$

where $X_W = \{x \in X : a_j(x) \in \mathcal{A},\ R+1 \in \mathcal{R}\}$ and $X_R = \{x \in X : a_j(x) \in \mathcal{A},\ R \in \mathcal{R}\}$, and where $x' = \left(a_j(x), \eta', R\right)$, $\eta' \in \mathcal{H}$, are the relevant pre-retirement decision states at age $j+1$. If Γ denotes the unique invariant distribution of Π, the initial household distribution is equal to

$$\Phi_\iota\left(\{0\} \times \mathcal{H} \times \{R\}\right) = \begin{cases} \int_{\mathcal{H}} d_\iota(0, \eta, \iota)\, \mathrm{d}\Gamma & \text{if } R = \iota + 1, \\ \int_{\mathcal{H}} \left(1 - d_\iota(0, \eta, \iota)\right) \mathrm{d}\Gamma & \text{if } R = \iota, \end{cases} \tag{33}$$

and zero for all other asset levels and retirement statuses. ◁

As in Romer (1990), the model features endogenous growth in total factor productivity Z_t through changes in the measure of intermediate varieties z_t. As in Jones (1995), this growth is semi-endogenous in that it is exogenously determined by the rate of population growth along a balanced growth path.

Definition 2 (Stationary equilibrium)**.** A stationary equilibrium, or steady state, is an equilibrium in which all variables grow at constant (possibly zero) rates and all growth rates are determined by the population growth rate $1+n \equiv \frac{N_{t+1}}{N_t}$. In particular, the growth rate of new intermediate-good firms is

$$1 + g_z = (1+n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} > 0. \tag{34}$$

The growth rates of TFP, output per capita, and hours per worker are similarly

$$1 + g_Z = (1+n)^{\gamma_Z}, \qquad 1 + g_y = (1+n)^{\gamma_y}, \qquad 1 + g_h = (1+n)^{\gamma_h},$$

where $\gamma_Z \equiv \chi \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}$, $\gamma_y \equiv \frac{1+\theta}{1+\theta\sigma} \gamma_Z$, and $\gamma_h \equiv \frac{\theta(1-\sigma)}{1+\theta\sigma} \gamma_Z = \gamma_y - \gamma_Z$. ◃

The growth rates in Definition 2 are derived in Online Appendix A. Note that the TFP growth rate reduces to Equation (2) in the simple model if we consider a steady state with constant hours worked (via log preferences, $\sigma \longrightarrow 1$) and a substitution parameter for intermediate goods exactly equal to the capital share parameter ($\rho = \alpha$).

4 Numerical Experiment and its Computation

As a quantitative exercise, I consider a transition from 1950 to 2100 where demographic change is the only exogenous driver. The transition mimics observed and projected population trends and eventually converges to a steady state consistent with the projected fertility, mortality, and migration rates by the end of the twenty-first century. To reduce the impact of the initial steady-state assumption on the period of interest, I simulate the model from a stationary equilibrium in 1900. In 1901, the economy is shocked by the demographic transition, after which agents have perfect foresight of aggregate variables until the new steady state. The 1901–1950 period serves as a calibration phase, during which model moments are matched to data to pin down key parameters. To isolate transitory growth effects, such as those from the baby boom or changes in old-age mortality, I impose a common population growth rate in both steady states. This ensures that GDP growth remains constant in the long run, allowing any deviations from this trend to be attributed solely to transitional dynamics.

Numerically, I solve the household problem using the discrete-continuous endogenous grid method introduced by Iskhakov *et al.* (2017) and approximate the household distributions with histograms over wealth, idiosyncratic productivity, and retirement status. Iskhakov *et al.* extend Carroll's (2006) endogenous grid method to incorporate discrete choices, which reduces the computational burden of modelling endogenous retirement. The overall model equilibrium is obtained via an Anderson acceleration procedure that iterates over paths of interest rates (r_t), pension contribution rates (τ_t^b), average labour supplies ($\bar{\ell}_t$), lump-sum transfers (tr_t), and growth rates of intermediate varieties (g_{zt}) until all equilibrium conditions hold.[12]

5 Bringing the Model to the Data

The model is calibrated to be consistent with key features of the US economy. To this end, I first set values for a subset of the model parameters based on exogenous estimations and previous literature. With these parameter values in hand, I then estimate the remaining parameters internally by matching transition path moments of the calibration phase (the pre-1950 period) with US macro data. The results are summarised in Table 3 and discussed in detail below.

[12] Anderson acceleration is an enhancement of fixed-point iteration that achieves superlinear convergence. Unlike comparable quasi-Newton methods, Anderson acceleration does not require an initial Jacobian matrix of derivatives, which makes it ideal for computationally heavy large-scale macroeconomic models.

Table 3. Calibrated parameters of the baseline model.

Parameter	Description	Value	Source or target
Households			
ι	Initial adult age	20	Children between 0–19
J	Maximum age	99	Certain death at 100
β	Discount factor	1.011	Capital-output ratio
ψ	Leisure weight	80.23	Hours per person aged 20–64
Ω_A	Bequest weight	16.55	Old-age asset wealth
σ	Inverse IES	1.75	Boppart and Krusell (2020)
θ	Frisch elasticity	0.5	Chetty *et al.* (2011)
ζ	Household consumption scale	1.49	Browning and Ejrnæs (2009)
$\{\omega_i\}_{i=0}^{\iota-1}$	Utility weights on dependants	Fig. F.4[a]	Browning and Ejrnæs (2009)
Individual productivity			
$\{\varepsilon_j\}_{j=\iota}^{J}$	Deterministic productivity	Fig. F.5[a]	PSID
φ	Persistence shock	0.97	Heathcote *et al.* (2010)
σ_ϵ^2	Variance shock	0.02	Heathcote *et al.* (2010)
Production			
α	Intermediate goods share	0.35	Labour share of income
ρ	Elasticity of substitution	0.67	Profit share of income
δ_k	Capital depreciation rate	0.034	Depreciation/output
R&D			
δ_z	Firm obsolescence rate	0.002	IPP depreciation/output
ν	R&D productivity	0.009	$z = Z = 1$ in initial period
λ	Duplication externality	0.75	Comin and Gertler (2006)
ϕ	Knowledge spillovers	−0.068	$g_Q = 5.97\,\% \implies g_Z = 1.11\,\%$
Social security			
μ	Replacement rate	0.413	Clingman *et al.* (2021)
R^{norm}	Normal retirement age	65	Social Security Administration
R^{min}	Lowest retirement age	62	Social Security Administration
$ps(R_j)$	Early/delayed scaling	0.8–1.15	Social Security Administration
Taxes			
τ^c	Consumption tax rate	0.080	BEA national accounts
τ^k	Capital gains tax rate	0.383	BEA national accounts
$\tau^w(w\bar{\ell})$	Income tax rate at mean income	0.116	BEA national accounts
κ_0	Asymptotic income tax rate	0.631	OECD tax database
κ_1	Income tax progressivity	0.574	OECD tax database
κ_2	Income tax scale parameter	0.505	OECD tax database
κ_3	Income tax rate at zero income	−0.205	OECD tax database

[a] Available in Online Appendix F.

5.1 Demographics

The United Nations (2024) World Population Prospects provides annual estimates on age-specific survival rates, age-specific fertility rates (births per woman), and aggregate net migration between 1950 and 2100. For earlier years, I collect age-specific survival rates and fertility rates from Bell and Miller (2005) and Heuser (1976), and use aggregate immigration data from the US Department of Homeland Security (2020) as proxy for aggregate net migration. The age distribution of migration is assumed throughout to match

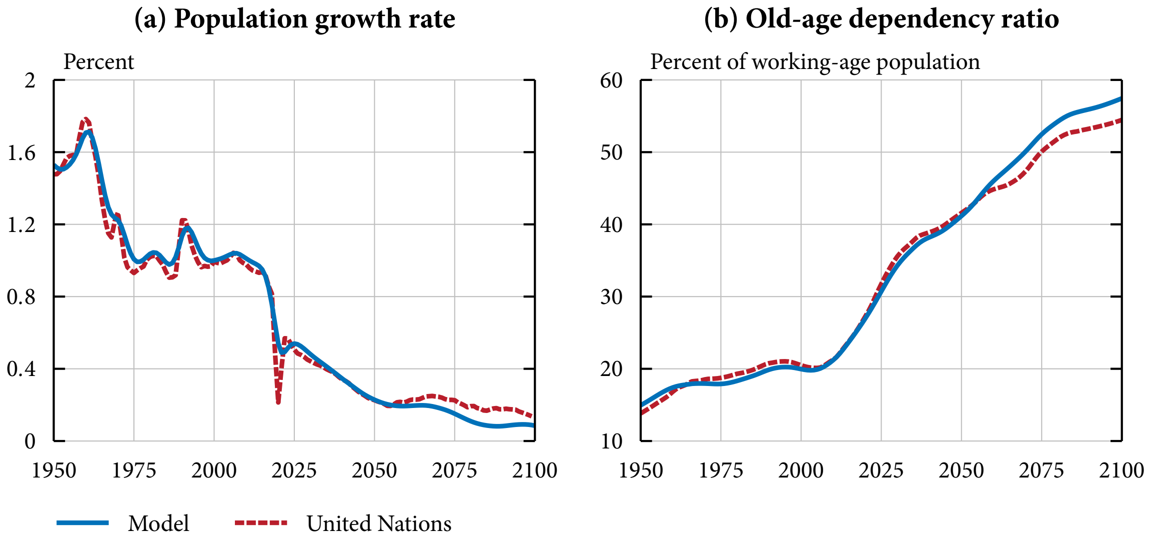


Figure 2. Demographic evolution.

the US Census Bureau's (2023) population projection. Fertility rates by age are only available from 1917, so for earlier years I adjust the 1917 rates by the change in the total fertility rate between these years and 1917 using estimates from the Gapminder Foundation (2024). The fertility rates and migration levels are then converted into births and migrants per person using population estimates from the Census Bureau's intercensal population tables and the UN World Population Prospects. Finally, I smooth all demographic variables with an HP-filter using an annual smoothing parameter of 6.25.

The resulting demographic variables for the year 2100 generate a steady-state population with near-zero growth. For simplicity, I marginally adjust migration rates by a common term so that population growth is exactly zero in the final steady state. I similarly impose a stable and constant population in the initial steady state which is consistent with the data on survival rates, fertility rates, and population size, again by adjusting migration rates with a common term.[13] Figure 2 plots the implied demographic development and shows that the calibration closely matches United Nations estimates.

5.2 Households

Households start their economic lives upon turning 20 and die with certainty at 100. I set the Frisch elasticity of labour supply θ to 0.5, as recommended by Chetty *et al.* (2011) along the intensive margin. This value is also consistent with Domeij and Flodén (2006a), who explicitly account for the bias arising from uninsurable income risk and borrowing constraints. The inverse intertemporal elasticity of substitution, σ, is calibrated following Boppart and Krusell (2020), who present macro evidence that 2 percent productivity growth in the long run implies a fall in hours worked by roughly 0.4 percent per year. Since the steady-state growth rates of hours worked and technology in Definition 2 are linked via $1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}$, this empirical pattern suggests that $\theta(1-\sigma)/(1+\theta\sigma) \approx -0.2$. Given $\theta = 0.5$, this generates a σ of 1.75. While based on long-run macro trends, these parameter values are also consistent with micro evidence: Heathcote, Storesletten and Violante (2014) consider an incomplete-markets model with similar preferences and estimate $(\sigma, \theta) = (1.71, 0.46)$ using US micro data.

[13] If x is a common migration rate shifter, a steady-state population satisfies $N_{j+1} = \frac{s_j+m_j+x}{1+n} N_j = \left(\prod_{k=0}^{j} \frac{s_k+m_k+x}{1+n}\right) N_0$. Consistency with fertility rates requires that $(1+n)N_0 = \sum_{j=0}^{J} f_j N_j$. Combining these yields the condition $1+n = \sum_{j=0}^{J} \left(\prod_{k=0}^{j-1} \frac{s_k+m_k+x}{1+n}\right) f_j$, which can be solved for x numerically.

The adult equivalence weights $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ and the household scale parameter ζ are based on Browning and Ejrnæs (2009), who estimate equivalence scales for low- and high-educated households that include age-specific weights for children and economies of scale. Browning and Ejrnæs use UK data, though Fernández-Villaverde and Krueger (2007) demonstrate with US data that a similar age-varying equivalence scale explains the hump shape of life-cycle consumption better than a traditional scale. The equivalence weights are set to the average of Browning and Ejrnæs' estimates for the two education groups. Browning and Ejrnæs' economies-of-scale parameter, call it ζ_{BE}, is linked to the scale parameter here via $\zeta_{BE}\,\sigma = \frac{\zeta-(1-\sigma)}{\zeta}$. Given $\sigma = 1.75$ and choosing $\zeta_{BE} = 0.86$, again the average of Browning and Ejrnæs' estimates, then yields $\zeta = 1.49$.

I estimate the deterministic age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ by a fixed-effects regression of real log wages on a quadratic in age using 1968–2019 Panel Study of Income Dynamics data (see Online Appendix B for details). The resulting profile features a standard hump shape that peaks between the ages of 40 and 50. The idiosyncratic productivity shocks are assumed to follow the AR(1) process $\log \eta' = \varphi \log \eta + \epsilon$, where $\epsilon \sim N(0, \sigma_\epsilon^2)$. The persistence parameter φ and error variance σ_e^2 set to 0.97 and 0.02, following the evidence in Heathcote, Storesletten and Violante (2010). This process is discretised into a five-state Markov chain using Rouwenhorst's method (Kopecky and Suen, 2010).

5.3 R&D Production Function

The R&D production function contains three parameters: the productivity level ν, the duplication externality λ, and the knowledge spillover ϕ. The first one is a scale parameter; I set it so that $z = Z = 1$ in the initial steady state. Bloom *et al.* (2020) highlight that there is no consensus on the correct value for λ, so I simply follow the calibrations in Jones and Williams (2000) and Comin and Gertler (2006) and set it to 0.75.

Like Jones and Williams (2000) and Bloom *et al.* (2020), I calibrate ϕ to make the R&D production function consistent with the time-series relationship between TFP and real gross investment in intellectual property products (IPP).[14] Figure 3, which plots data from the Penn World Table and the Bureau of Economic Analysis (BEA, NIPA Table 5.2.3), shows that these variables exhibit roughly constant growth rates of 1.11 and 5.97 percent per year since the 1950s. To be consistent with these trends, the model R&D equation $1 + g_z = 1 - \delta_z + \nu Q^{\lambda} z^{\phi-1}$ must generate stable growth in which the last term on the right-hand side remains constaint whenever z and Q grow at similar constant rates. Assuming this condition holds, log-differentiating the last term and substituting the TFP definition $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$ then yields the calibration equation $\phi = 1 - \lambda \frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}\frac{g_{IPP}}{g_{TFP}}$, where g_{IPP} and g_{TFP} are the growth rates from Figure 3. Given the baseline values for λ, α, and ρ, this results in $\phi = -0.068$.[15] This calibration, in turn, implies steady-state growth exponents γ_Z and γ_y of 0.22 and 0.18, close to the mid-range of Jones's (2002) estimates.[16]

[14] Besides formal R&D, IPP investment includes nonrival goods such as computer software and entertainment, literary, and artistic originals. This broader measure is arguably closer to the model's notion of innovation, so I use total IPP investment rather than formal R&D as the relevant calibration variable.

[15] This value is larger than what Bloom *et al.* (2020) finds for the aggregate US economy. However, Bloom *et al.* estimate ϕ under the implicit assumption that $\rho = \alpha$. Imposing the same restriction here yields a similar value.

[16] A steady state with $(\gamma_Z, \gamma_y) = (0.22, 0.18)$ and for example 1.2 percent annual population growth exhibits TFP and output per capita growth of 0.26 and 0.21 percent per year.

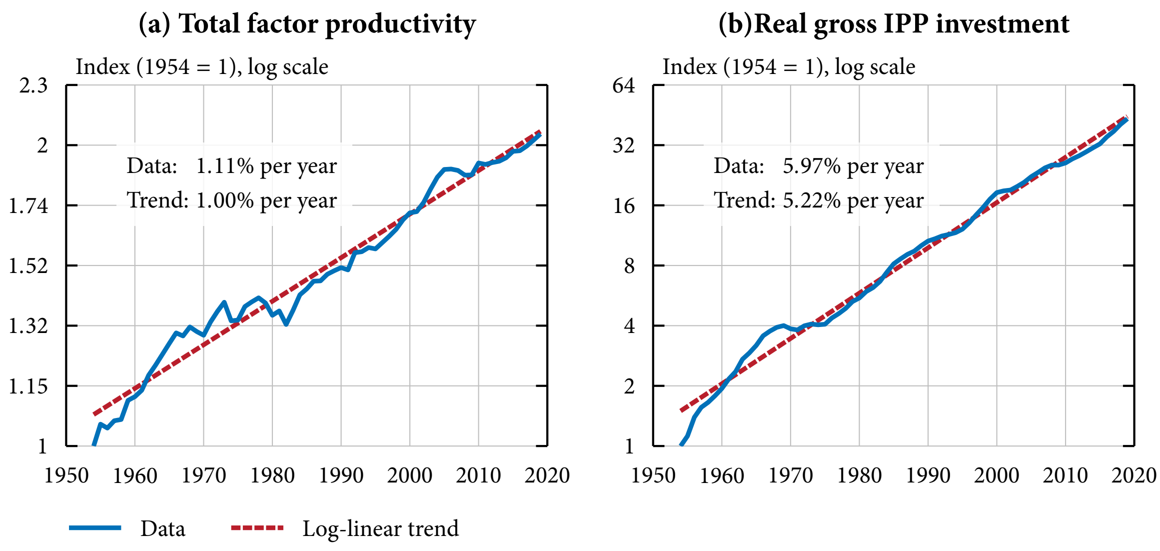


Figure 3. US TFP and aggregate IPP investment in the data.

Source. TFP from the Penn World Table (Feenstra, Inklaar and Timmer, 2015), expressed as Harrod-neutral technological change using labour shares from the same source. IPP investment from BEA NIPA Table 5.2.3.

5.4 Public Sector

I set the gross pension replacement rate μ to 0.413 based on Clingman, Burkhalter and Chaplain's (2021) estimates for average-income workers who retire at the normal retirement age. The normal retirement age, R^{norm}, is 65 and the earliest age to collect retirement benefits, R^{min}, is 62. Early and delayed retirement adjustment via $ps(R)$ is similar to that of the US Social Security system. For every year of early retirement, the base level benefit is reduced by 6 ⅔ percent per year for the first three years and 5 percent per any additional year. For every year of delayed retirement, the base level benefit is scaled up by 3 percent until a maximum age of 70. After 70, no extra benefit is given for delaying retirement.[17]

The income tax rate $\tau^w(e)$ at household earnings e is parametrised by the Gouveia and Strauss (1994) form:

$$\tau^w(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3, \tag{35}$$

where κ_0 controls the asymptotic tax rate $\lim_{e\to\infty} \tau^w(e) = \kappa_0 + \kappa_3$, κ_1 determines the degree of tax progressivity, and κ_2 is a scale parameter. Equation (35) augments Gouveia and Strauss' original function with average labour earnings $\bar{e}$, which makes the tax function invariant to units of measurement, and the parameter κ_3, which allows for a non-zero marginal tax rate at zero earnings.

To calibrate the taxes, I first construct tax rates on consumption, capital, and labour income using national accounts data from the BEA by dividing the aggregate revenues of each tax with its corresponding tax base. Taking the average of each tax rate between 1950 and 2022 gives τ^c, τ^k, and $\tau^w(\bar{e})$, the latter being the income tax rate at average earnings. Next, I estimate $\{\kappa_0, \kappa_1, \kappa_2, \kappa_3\}$ using the OECD tax database, which provides income-specific tax rates that incorporate federal, state, and local government taxes plus several deductions and credits. These estimates are available annually since 2000, but only for a few representative income levels. I therefore replicate the OECD's methodology to compute tax rates over a full grid of

[17] These are the exact retirement ages and scaling rules used by the Social Security Administration for cohorts born before 1924. Later cohorts have higher normal retirement ages and more generous delayed retirement credits, though Online Appendix E shows that the quantitative results are insensitive to a more accurate development of R^{norm} and $ps(R)$.

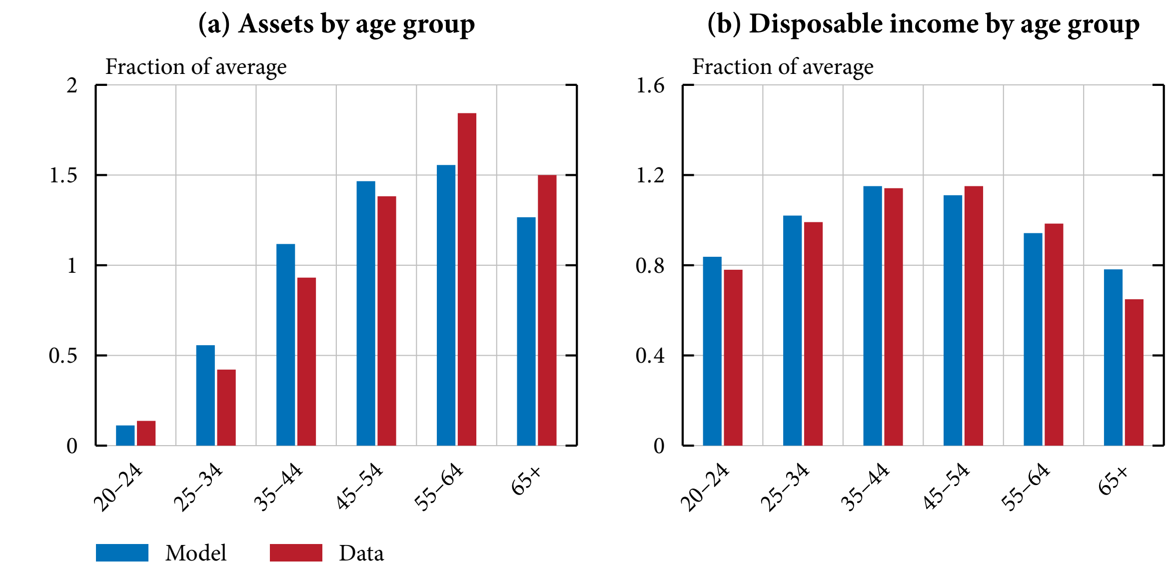


Figure 4. Life-cycle profiles in 1950 in the model and the data.

Source. Net worth from the 1949 Survey of Consumer Finances (Goldsmith, 1955, Table XLIII). Disposable income from the 1950 Survey of Consumer Expenditures (US Bureau of Labor Statistics, 1957, Table 1-1).

incomes for each year and fit (35) to these estimates. Finally, for consistency with the aggregate data, I shift the level of the tax curve by adjusting the estimated parameters until the tax rate at average earnings, $\tau^w(\bar{e})$, equals the tax rate obtained from the national accounts. Online Appendix C outlines all the details of these steps.

5.5 Internally Calibrated Parameters

Seven parameters are estimated using the simulated method of moments procedure for Auerbach and Kotlikoff (1987) models proposed by Ludwig (2005): the discount factor (β), the utility weights on labour supply and bequests (ψ, Ω_A), the depreciation rates for capital and R&D (δ_k, δ_z), and the production function parameters (α, ρ). The calibration aligns model-generated moments from the pre-1950 calibration phase of the transition path to their empirical counterparts over the same period, thus ensuring consistency with historical data. This estimation and the computation of the transition path are consequently performed simultaneously, a computationally intensive and time-consuming task.

In order of the parameters above, the matched moments and their sample years are: the capital-output ratio (1901–1949), hours worked per person aged 20–64 (1901–1949), the average net worth of individuals aged 65+ relative to those aged 55–64 (1950), consumption of non-IPP capital relative to GDP (1901–1949), consumption of IPP capital relative to GDP (1929–1949), and the labour and profit shares of national income (1901–1949). Given the lack of official US data for first half of this period, I construct the empirical moments by splicing historical sources with BEA and Bureau of Labor Statistics data. One exception is IPP depreciation, which, to my knowledge, does not exist before the BEA series, thus restricting this moment to the 1929–1949 subperiod. Online Appendix D provides further details and shows the fit of the model-generated time series to their empirical counterparts.

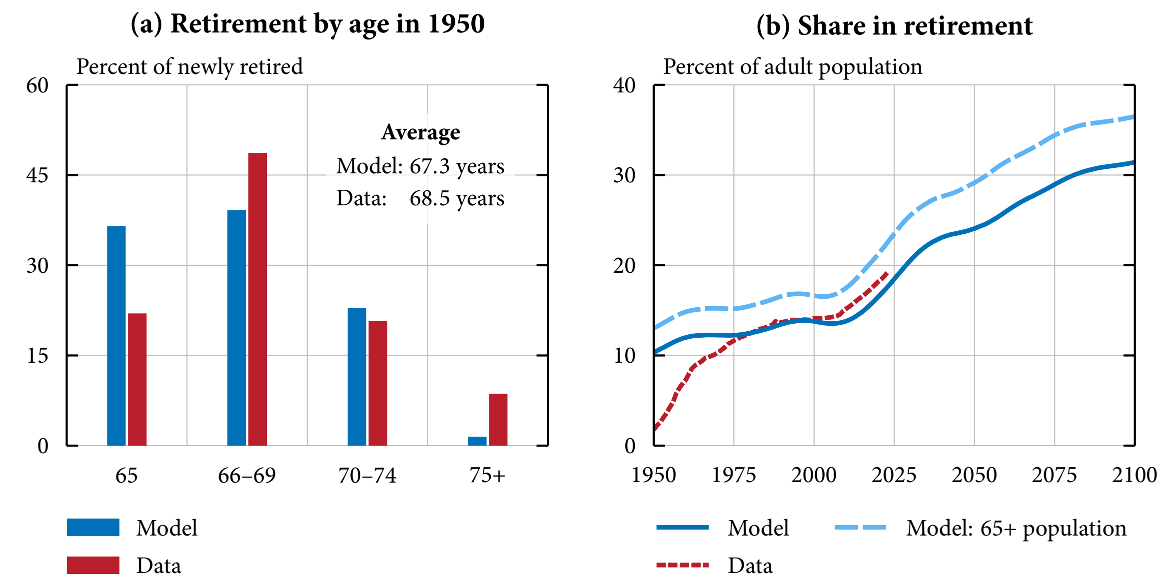


Figure 5. Retirement in the baseline model and the data.

Source. Social Security Administration (2024, Tables 5.B5 and 6.B5).

5.6 Is This an Empirically Realistic Model?

Lastly, before turning to the main results, I examine how key predictions from the baseline calibration compare to nontargeted data along the computed equilibrium transition path.

Figure 4 compares predicted and empirical cross-sectional age profiles of assets and after-tax income at the onset in 1950. The fit of the asset profile is reasonable, and the calibrated bequest motive ensures that the relative difference between peak and retirement ages is consistent with the data. Yet, discrepancies arise as the model generates too much saving at younger ages. This is a consequence of the discount factor being larger than one, which increases the weight households place on future consumption. A more realistic cross-sectional profile of assets (and consumption) requires a β less than one, but such a discount factor is inconsistent with the capital-output ratio being targeted in the calibration. Given the research question of the paper, the latter is given priority. This choice and the resulting value of β is also consistent with the seminal paper of Hurd (1989), who presents empirical support for precisely $\beta = 1.011$ when accounting explicitly for mortality risk. No such tension exists for income, where the model provides an excellent fit of the data.

What about retirement behaviour? While we should neither expect nor demand the stylised retirement framework to perfectly replicate the data, Figure 5a shows that it aligns well with cross-sectional retirement patterns in 1950. At that time, however, Social Security eligibility was limited; as the program expanded, the average retirement age declined—a pattern not reflected in the model. Nevertheless, Figure 5b reveals that from the 1970s onward—when Social Security approached universal coverage—the model closely tracks the share of retired workers in the data. The difference in earlier years occurs precisely because the model assumes universal coverage and constant eligibility rules throughout. While counterfactual, this assumption eliminates policy-induced variation in the model, thus providing a cleaner numerical experiment focused solely on demographic change. The predicted retirement share also closely mirrors the fraction of the population aged 65 and above, which suggests that the model at minimum performs as well as the benchmark case without endogenous retirement, where such a trend is imposed by assumption.

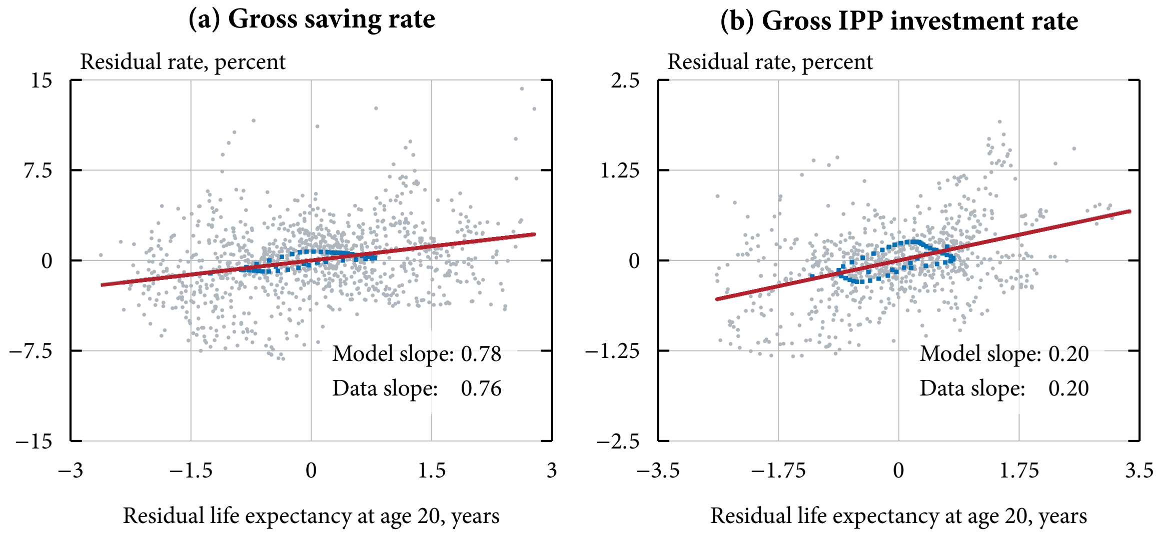


Figure 6. Partial correlations of savings, investment, and life expectancy: model and data.

Notes. The blue squares are predictions from the model between 1970 and 2020, after partialling out population dependency rates (0–14 and 65+ population shares). The red line shows the OLS fit. The gray elements are the OECD data in Figure 1.

Ultimately, the cross-sectional profiles are of secondary importance as long as the predictions for the key mechanisms remain reasonable over time—not only for the share of people in retirement but also the aggregate saving rate and, in particular, the R&D investment rate. To assess the model along these dimensions, I replicate Figure 1 using gross saving and investment rates from the model's baseline transition path for comparable years. Figure 6 plots the results against the original data from Figure 1. Between 1970 and 2020, the model predicts that a one-year increase in life expectancy at age 20 raises the gross saving rate by 0.78 percentage points and the gross R&D investment rate by 0.20 percentage points, conditional on the population age structure. These estimates are identical to the overall OECD data, thus supporting the model's ability to accurately capture the core channels driving the results in the next section.

6 Quantitative Results

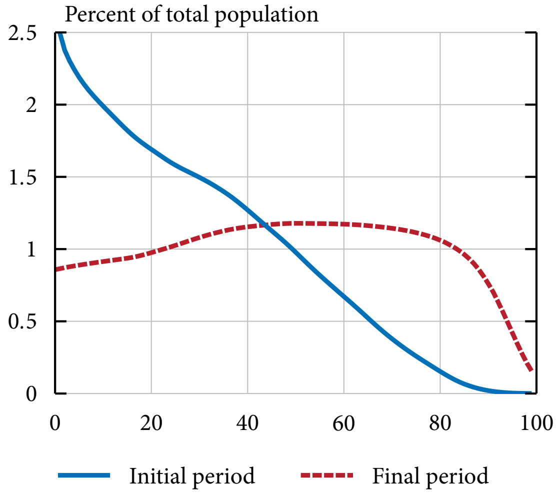
The lack of population growth in the initial and final steady states means that there is no long-run economic growth. The growth impact of the demographic transition is therefore purely transitory. This section illustrates and quantifies this impact and its underlying mechanisms via three quantitative exercises: (i) a growth accounting analysis of per-capita growth in the baseline model, (ii) counterfactual simulations that identify the key demographic factors at work, and (iii) a comparison to an equivalent model without endogenous growth to assess the importance of the TFP channel.[18] Throughout, I take "per capita" to mean "per adult equivalent person", since children only affect the economy through the household size.

The growth accounting exercise decomposes aggregate labour—the total efficiency units supplied by households—into total employment (E), average hours per worker ($\bar{h}$), and average productivity per hour worked ($\bar{\varepsilon}$): $L = E\,\bar{h}\,\bar{\varepsilon}$. Combining this with output per capita, $y = Z\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}\frac{L}{N}$, and log-differencing between subsequent periods yields the growth accounting identity

$$g_y = g_Z + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + g_h + g_\varepsilon, \tag{36}$$

[18] Additional figures of other variables are available in Online Appendix F.

(a) Population age distribution



(b) Cumulative transitory growth

$$y = Z \cdot \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{E}{N} \cdot \bar{h} \cdot \bar{\varepsilon}$$

Output per capita	TFP	Capital intensity	Emp. rate	Hours	Prod.
45%	72%	17%	−22%	−10%	3%
(100%)	(160%)	(38%)	(−49%)	(−22%)	(6%)

Figure 7. Comparative statics: population structure and output per capita.

Notes. Figure 7b shows the cumulative growth in output per capita and each of its contributing factors along the demographic transition. The numbers in parentheses displays their relative contributions to overall growth.

where g_x denotes the net growth rate of a variable x. Equation (36) splits the change in output per capita into changes in technology, the capital intensity, the employment rate, average hours per worker, and average productivity per hour worked, which makes it easy to quantify the transitional mechanisms at work.[19]

6.1 Comparative Statics

Following the discussion in Section 2, it is useful to first consider comparative statics of the baseline scenario. Figure 7 displays the overall change in the age structure and the cumulative percentage change in output per capita between the initial and final steady states, which reveals that the change in the population structure raises output per capita by 45 percent. This follows primarily from a substantial increase of 72 percent in TFP relative to its initial level. The capital intensity also increases by 17 percent while the higher share of old-age households in the population reduces the employment rate by 22 percent. The qualitative changes in the underlying channels are therefore precisely as the simple model predicts.

A natural question immediately emerges from Figure 7: why is the magnitude of the change in TFP so much larger than any of the other changes? It is not because an overwhelming majority of household savings are allocated to R&D rather than to capital investments.[20] Rather, this is the effect of the endogenous growth scale effect from Section 2, which occurs when the demographic transition increases the population size. A simple decomposition of TFP illustrates this point. Specifically, per Definition 2 we can exploit that TFP is proportional to the size of the labour force E raised to γ_Z in steady state. The corresponding proportionality constant provides a measure of the R&D intensity, just as in Equation (4) of the simple model. The scale effect is pinned down by the overall change in employment, E_T/E_0, where 0 and T denote

[19] Generally, for any fixed long-run population growth rate $\tilde{n}$, we can identify and decompose transitional growth as in Jones (2002) via $g_y - \gamma_y \tilde{n} = (g_Z - \gamma_Z \tilde{n}) + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + (g_h - \gamma_h \tilde{n}) + g_\varepsilon$.

[20] Capital investment exceeds R&D investment by a factor of 4 during most of the transition and the growth rate of the capital stock relative to TFP is even higher; see Figure F.6 in Online Appendix F.

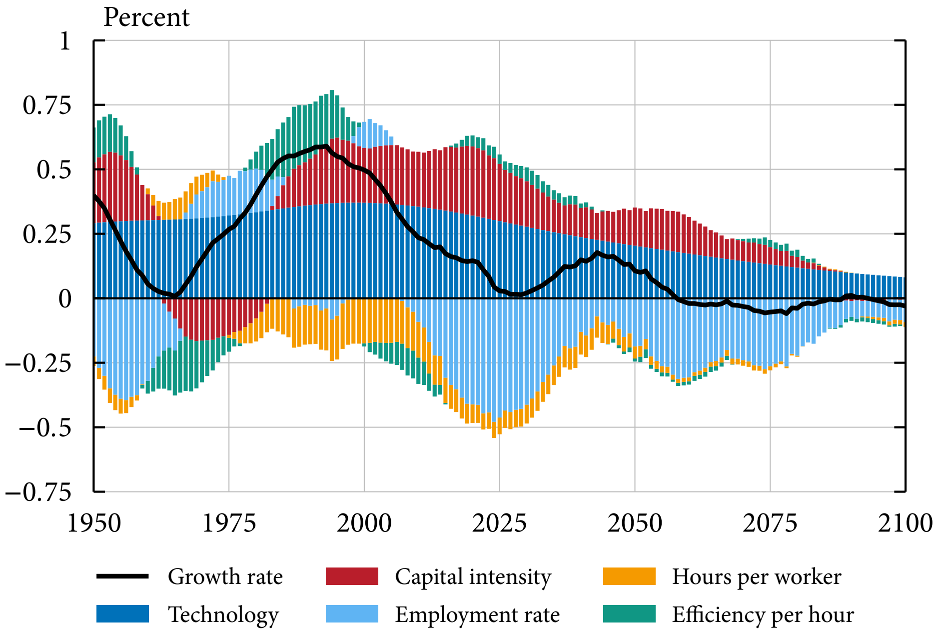


Figure 8. Growth accounting of the baseline model.

the initial and final steady states. We then obtain the following:

$$\underset{\uparrow 72\,\%}{\text{TFP change}} = \frac{Z_T}{Z_0} = \frac{Z_T \,/\, E_T^{\gamma_Z}}{Z_0 \,/\, E_0^{\gamma_Z}} \cdot \left(\frac{E_T}{E_0}\right)^{\gamma_Z} = \underset{\uparrow 18\,\%}{\begin{array}{c}\text{Change in}\\ \text{R\&D intensity}\end{array}} \times \underset{\uparrow 47\,\%}{\text{Scale effect}}\,,$$

where the percentages show the cumulative change in each factor between the initial and final steady states. The 72 percent rise in TFP is due to a 18 percent increase in the R&D intensity and a 47 percent increase in scale. Changes in household behaviour and composition thus impact output per capita through the R&D intensity much like they do through the capital intensity. The difference between TFP and the other factors of production instead lies in the fact that technology is nonrival, as seen from the large scale effect.[21]

6.2 Growth Dynamics

Comparative statics are illustrative but says little about the dynamics during the transition. Therefore, we now turn to the period of interest: 1950 to 2100. Figure 8 plots the growth rate and its corresponding decomposition (that is, the right-hand side of (36)) during this period. Focusing first on the overall growth rate, three key developments emerge. First, the demographic transition positively affects output per capita throughout the second half of the twentieth century, with growth firmly above zero. Then, this effect fades after the turn of the century, causing a significant drop in the growth rate. Finally, despite this decline, the demographic development does not negatively affect output per capita. Instead, growth remains around zero throughout the twenty-first century.

Looking at the underlying mechanisms, we again find a qualitative development as predicted in Section 2: the demographic transition generates positive TFP growth and capital deepening, but lowers the employ-

[21] A caveat here is that using employment as the relevant scaling variable is somewhat arbitrary. We could just as well use the overall population size, since it grows in parallel with employment in steady state. Switching to the population size (measured in adult equivalents) yields a slightly larger scale effect, 55 percent, although this does not change the basic point of the exercise.

Table 4. Growth accounting of the baseline model.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
1950–2100	0.16 %	0.25 %	0.11 %	−0.15 %	−0.06 %	0.01 %
1950–2000	0.33 %	0.33 %	0.07 %	−0.05 %	−0.06 %	0.04 %
2001–2100	0.08 %	0.21 %	0.13 %	−0.20 %	−0.05 %	0.00 %
1995	0.55 %	0.37 %	0.25 %	−0.07 %	−0.17 %	0.16 %
2030	0.02 %	0.28 %	0.18 %	−0.41 %	−0.07 %	0.05 %
Difference	−0.53 pp.	−0.09 pp.	−0.08 pp.	−0.34 pp.	0.10 pp.	−0.11 pp.

Notes. The table reports average annual growth rates according to the growth decomposition in Equation (36). Individual growth rates may not sum to totals due to rounding.

ment rate. Moreover, average hours worked declines in periods of positive growth (and vice versa) while average efficiency per hour worked rises in periods where the share of middle-aged workers is higher. The former is due to the parametrisation of household preferences, where the income effect of higher wages on leisure dominates the substitution effect, while the latter follows from the hump shape in workers' life-cycle productivity profile.

Table 4 quantifies these observations by summarising the average annual growth rates of output per capita and its components, first for the entire period considered and then separately for each century. Overall, the demographic transition boosts output per capita by 0.16 percent per year. The effect is primarily driven by the twentieth-century development, where the average annual growth rate is 0.33 percent. By comparison, the observed long-run growth rate of GDP per person in US data is approximately 2 percent per year. From this viewpoint, the impact obtained here is quantitatively significant; taken at face value it implies that over 15 percent of actual US post-war growth can be attributed to transitory demographic factors. This contribution makes demographics comparable in importance to rising educational attainment, whose contribution US growth over the same period is also around 20 percent according to estimates by Fernald and Jones (2014).

Just as in the comparative statics, what stands out quantitatively from both Figure 8 and Table 4 is the importance of TFP. Over the full period, TFP grows by 0.25 percent per year, thus accounting for more than 160 percent of economic growth. Its contribution is about two and a half times as large as that of capital deepening. The effect is even larger during the twentieth century, with TFP growing by 0.33 percent per year, about five times faster than the capital intensity, thus driving the bulk of output growth.

A declining share of workers in the population constitutes the main drag on output growth, depressing the annual growth rate by on average 0.15 percentage points overall and by 0.20 percentage points during the twenty-first century. This happens as the average retirement age does not keep up with increasing life expectancy; recall Figure 5b, which shows that although there is a small shift towards later retirement over time, the increase in the retirement age is too marginal to meaningfully increase the gap between the share of retired workers in the population and the share of people aged 65 and above.

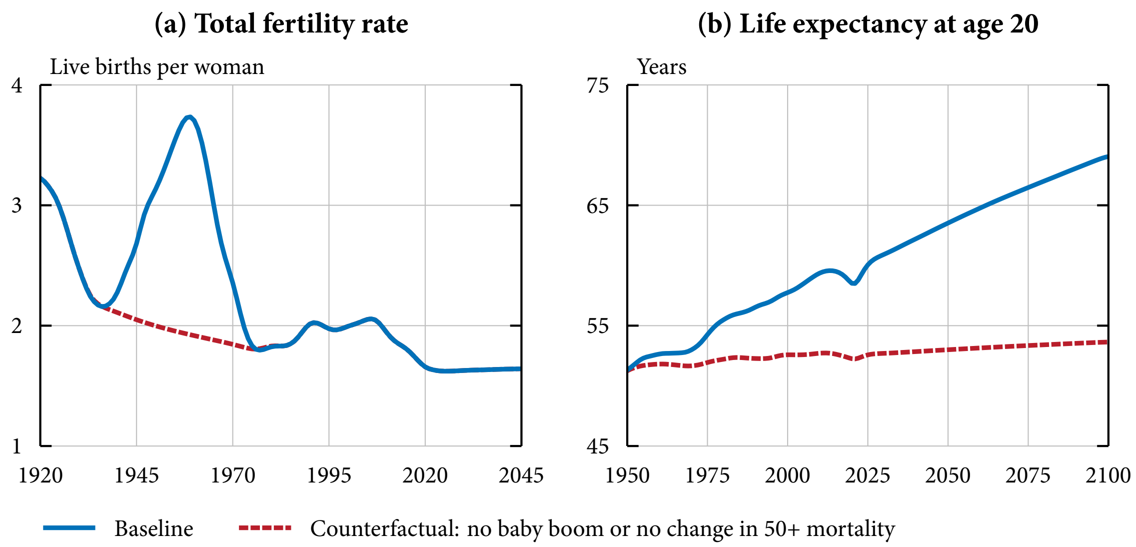


Figure 9. Demographic counterfactual scenarios.

Notes. The total fertility rate and the life expectancy are computed under the assumptions that the fraction of women in an age group and the average fraction of the year lived at the age of death remain the same as in official data and projections.

Due to concerns about recent declines in observed growth rates, it is also interesting to zoom in on the growth decline from peak in 1995 to trough in 2030. The decomposition for this period is shown in the lower half of Table 4. The growth rate and most of its components change monotonically between these years, so it suffices to consider snapshots at the beginning and end of this period and the differences between them. Overall, changes in the demographic structure lead to a 0.53 percentage point drop in the growth rate over the last three decades, thus suggesting that demographics exerts significant downward pressure on contemporary growth rates. The decline stems in part from roughly similar declines of 0.1 percentage points in the growth rates of TFP, capital intensity, and average efficiency. The growth rate of hours increases by a similar magnitude, therefore marginally counteracting the overall development. The majority, however, comes from the retirement of the baby boom: growth in the employment rate declines by 0.34 percentage points, accounting for 65 percent of the total decline.

6.3 Is It All About Boomers? Identifying the Demographic Effects

Related to the last point above, it appears at first sight that Figure 8 just reflects the baby boom dynamics predicted in Section 2. The share of young workers increases when the baby boomers enter adulthood in the 1960s and 1970s, which improves the employment rate and worsens the capital intensity and average productivity. When these cohorts become middle-aged in the 1980s and 1990s, growth through TFP, the capital intensity, and average productivity improves. Finally, they retire in the early decades of the twenty-first century, thereby causing a negative growth impact via the employment rate. Yet, Section 2 also stresses the increase in life expectancy from rising middle- and old-age survival rates. Which of these forces, if any, is the key driver of the results above?

To answer this question, I consider three counterfactual scenarios. First, I simulate the model under the assumption that neither the baby boom nor the rise in survival rates happens. That is, I replace fertility rates between 1935 and 1975 with interpolated values and hold survival rates above the age of 50 fixed from 1950 onward. As shown in Figure 9, these adjustments remove the hump shape in fertility associated with the baby boom and almost all life expectancy improvements after 1950. I then switch either fertility or mortality back to the baseline calibration and look at the change relative to the baseline model. Without

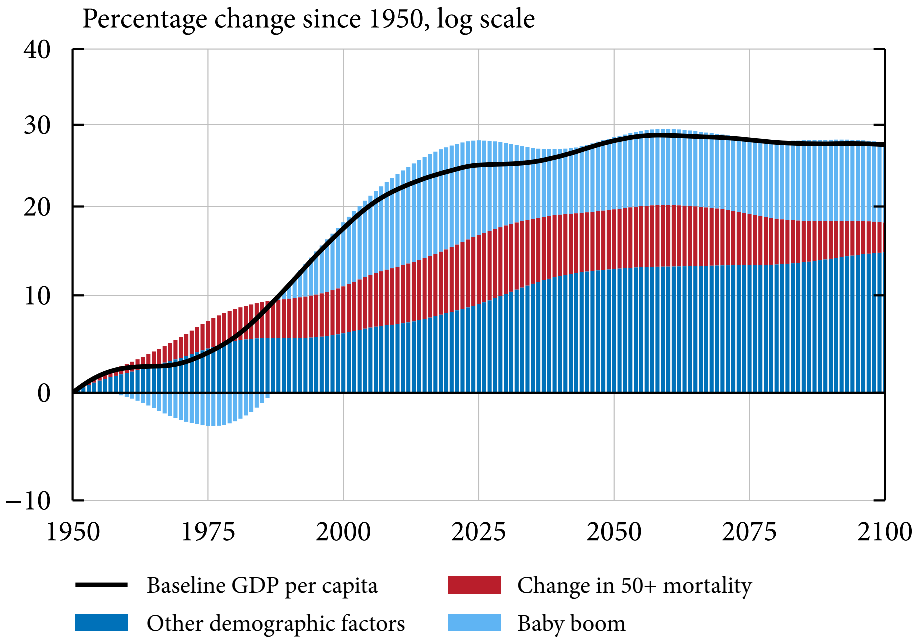


Figure 10. Identifying the impact of the demographic forces.

strong interaction effects, these scenarios isolate the impacts of the baby boom, of the change in middle- and old-age mortality, and of all other demographic changes.

Figure 10 plots the resulting decomposition of the cumulative change in output per capita into each demographic factor. While the growth swings in Figure 8 are clearly attributed to the life-cycle phases of the baby boomers, they are by no means the sole explanation behind the overall development. Between 1950 and 2100, output per capita rises by a total of 27.5 percent, which is accounted for by an 8.2 percent increase due to the baby boom, a 3.0 percent increase due to middle- and old-age mortality changes, and a 14.7 percent increase due to other demographic factors. The baby boom and changing mortality consequently explain around half of the baseline results, with remaining demographic changes accounting for the rest.

Table 5 displays the demographic contributions to each component of the average growth rate between 1950 and 2100. This shows that the baby boom and changing mortality generate around a quarter each of the total technological progress. Transition dynamics notwithstanding, this TFP contribution (and the subsequent income effect on hours worked) is the baby boom's only impact in the end, thus confirming the prediction from the simple model. This is unsurprising since the counterfactual fertility rates affect the cohorts that become adults after 1955 and die with certainty no later than 2075. Before and after these years, the age structure of the adult population is nearly identical to the baseline. Most changes in the capital intensity and the employment rate are instead driven by life expectancy improvements. Despite its positive impact on output per capita, longer lifespans also increase hours per worker, reflecting households' need to finance longer retirement.

What explains the residual factor? First, the population in 1950 is not stationary. Changes in the population structure therefore occur even without further changes in the underlying demographic variables, thus impacting the saving rate, the employment rate, and the average productivity per hour worked. This shift in the age structure is largely complete by the end of the twentieth century. Additionally, the changes in

Table 5. Growth decomposition of the demographic forces.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
Average growth rate	0.16 %	0.25 %	0.11 %	−0.15 %	−0.06 %	0.01 %
Baby boom	0.05 pp.	0.07 pp.	0.00 pp.	0.00 pp.	−0.02 pp.	0.00 pp.
50+ mortality	0.02 pp.	0.06 pp.	0.05 pp.	−0.12 pp.	0.03 pp.	0.00 pp.
Other changes	0.09 pp.	0.13 pp.	0.05 pp.	−0.04 pp.	−0.07 pp.	0.01 pp.

Notes. The table reports average annual growth rates between 1950 and 2100 according to the growth decomposition in Equation (36) and the percentage point contributions by each demographic factor. Individual growth rates may not sum to totals due to rounding.

fertility, mortality, and migration rates that nevertheless occur raise the population size, so the scale effect on TFP is also at work here, as seen from the large impact on TFP growth.

6.4 Is It All About Technological Progress? The Exogenous Growth Case

The finding that the demographic transition does not negatively impact per-capita output contrasts with the general notion of population ageing as a major drag on economic activity. For instance, Krueger and Ludwig (2007), the perhaps closest paper to the analysis here, use a similar quantitative model and find a 12.6 percent cumulative drop in US output per capita between 2005 and 2080 due to the demographic transition. However, most previous work, including Krueger and Ludwig, ignore the TFP channel. Another important question is thus to what extent this additional mechanism explains the difference.

As a final exercise, I therefore analyse a version of the model without technological change. Specifically, the benchmark model nests a standard model without endogenous growth as the special case with perfect substitution between intermediate firms ($\rho = 1$) and a zero intermediate-firm exit rate ($\delta_z = 0$). The former eliminates profits, thus forcing the patent price and R&D investment to zero, which in turn reduces the intermediate-firm dynamics to $z_{t+1} = (1 - \delta_z)z_t$. The latter ensures that the measure of intermediate firms remains constant over time. I also recalibrate the structural parameters β, ψ, Ω_A, and δ_k here to maintain the baseline calibration targets.

Figure 11a plots the growth rate under this specification against that of the baseline model. The positive effects now disappear, with growth rates consistently around 0.1 to 0.4 percentage points below the baseline. The cumulative decline in output per capita is nearly identical to Krueger and Ludwig (2007): 12.1 percent between 2005 and 2080 and 12.2 percent over the full period. In comparison, for the same periods the baseline model exhibits positive cumulative growth of 6.3 and 27.5 percent, respectively. The difference is driven almost entirely by the TFP difference, as shown in Figure 11b. The impacts on the capital intensity, the employment rate, and the average efficiency are virtually unchanged.[22] There is also a small counteracting effect in hours worked: lower income leads to a rise in hours, and this raises output. But this effect is too small to offset the lack of technological progress. Thus, whether the demographic transition raises or lowers output per capita turns out to depend crucially on whether or not we account for its impact on technological change.

[22] See Figure F.7 in Online Appendix F, which fully decomposes the growth rate gap.

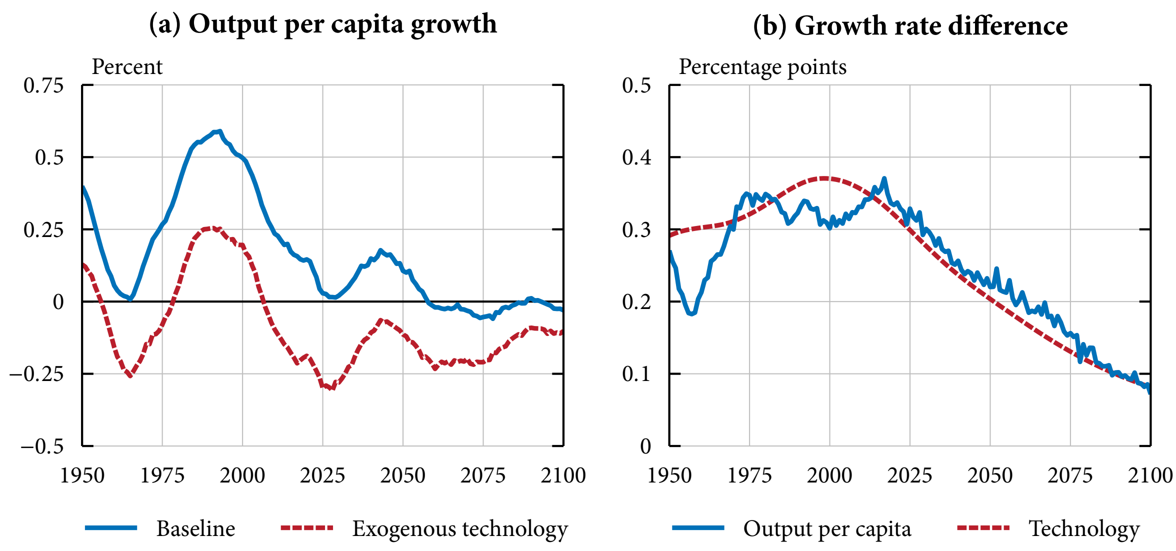


Figure 11. Comparing growth with and without endogenous technological change.

6.5 Taking Stock

In sum, neither the comparative statics nor the transition dynamics of the model suggests that the demographic transition, and population ageing in particular, is detrimental to economic growth once we account for its effect on technological progress. If anything, the impact is (temporarily) positive. Although demographics create significant downward pressure on growth rates over the last three decades due to a rising share of retirees in the population, it is important to stress that this is *not* because population ageing is particularly bad for economic output. In fact, it is hardly due to population ageing at all. Rather, the temporary rise in births that led to the baby boom generates a long period of above-normal growth in the last quarter of the twentieth century, when these generations were of prime working age. We are just now experiencing the end of that period as these generations retire. As already stressed in the simple model, the subsequent growth decline would occur even in the absence of improving life expectancy.

7 The Importance of the R&D Production Function

A central element in the results above is the R&D process, and this section closes the paper by examining the model's sensitivity to two critical components: the knowledge spillover parameter ϕ and the specification of the R&D production function. The former is particularly important, since ϕ determines the extent to which R&D becomes easier or harder over time, and its true value is likely subject to considerable uncertainty due to the challenges in measuring productivity growth and R&D investment. Additional robustness checks, detailed in Online Appendix E, include (i) gradually raising the normal retirement age as scheduled by the Social Security Administration, (ii) removing idiosyncratic income risk, (iii) eliminating the bequest motive, (iv) adopting a flat income tax, (v) imposing log preferences on consumption to fix long-run hours worked, and (vi) introducing a part-time wage penalty on hours worked. None of these adjustments meaningfully alter the benchmark results.

To assess the role of knowledge spillovers, I vary ϕ to ensure that the growth exponent γ_y aligns with the upper and lower bounds identified by Jones (2002). For weaker spillovers, I set $\phi = -1.4$, for which γ_y equals 0.07. This is also the value of ϕ found by Bloom *et al.* (2020) for the aggregate US economy when $\lambda = 0.75$. For stronger spillovers, I set $\phi = 0.4$, which gives $\gamma_y = 0.36$. To evaluate the specification of the

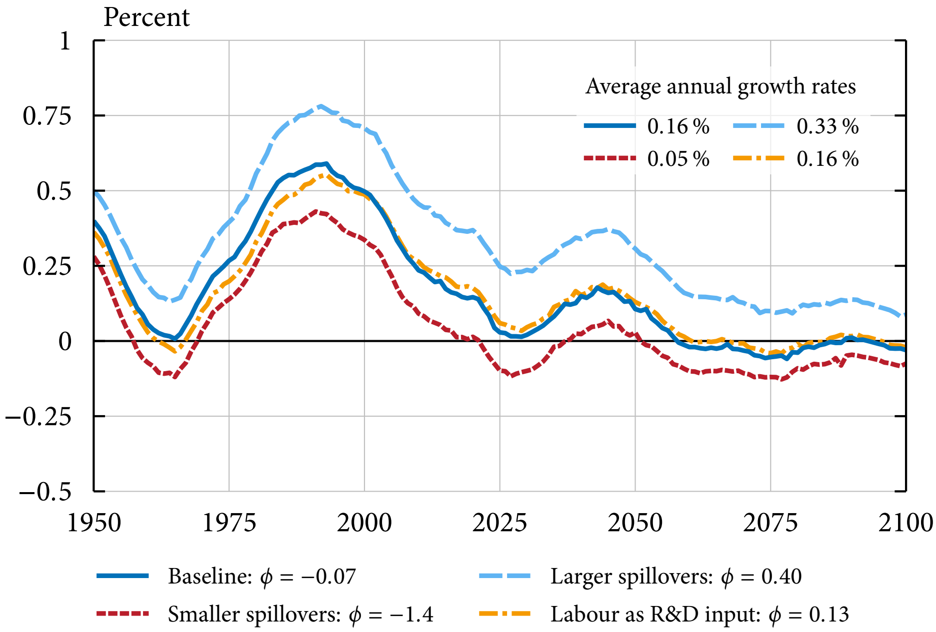


Figure 12. Growth rates with different degrees of knowledge spillovers and R&D inputs.

R&D production function, I consider an alternative commonly used in the growth literature, where labour rather than final goods serves as R&D input:

$$z_{t+1} \;=\; (1-\delta_z)z_t \;+\; \nu L_{zt}^{\lambda} z_t^{\phi},$$

where L_{zt} represents total R&D labour.[23] For comparability, I set $\phi = 0.13$ to match the long-run growth rate of the baseline model. In all cases, I again re-estimate the internally calibrated parameters to maintain consistency with the baseline calibration targets.

Figure 12 displays the growth rates from these alternatives against that of the baseline. Qualitatively, changing the knowledge spillover is straightforward: the larger the knowledge spillover, the larger the growth impact from R&D, since researchers become more productive the more knowledge is created. Figure 12 confirms this prediction. Meanwhile, changing the R&D input from final goods to labour generates a higher demand for labour relative to capital from producers, with subsequent general equilibrium effects on factor prices. Since the R&D sector is a small share of the total economy, these effects are minor. None of the key mechanisms of the model changes here, as the cost of R&D (the wage bill of researchers in this case) and the patent purchases by intermediate firms are still financed via household savings. The impact relative to the baseline is therefore negligible.

Growth rates are overall positive across the board (albeit quantitatively insignificant with small knowledge spillovers). Relative to the baseline, the average annual growth rate between 1950 and 2100 decreases and increases by about 0.1 to 0.15 percentage points with the different degrees of knowledge spillovers and remains unaffected when labour is the sole R&D input. Only in the most pessimistic calibration do we observe a negative impact on per-capita output during the twenty-first century, but even here the impact

[23] The labour and goods market conditions also become $L_t + L_{zt} = \sum_j N_{jt} \int_X \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}$ and $Y_t + A_{t+1}^M = C_t + G_t + \left[K_{t+1} - (1-\delta_k)K_t\right]$. As shown in Online Appendix A, long-run growth again follows the population growth rate via $1 + g_z = (1+n)^{\chi}$, but now with $\chi \equiv \frac{\lambda}{1-\phi-\lambda\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$.

is small at about −0.02 percent per year. Therefore, although reasonable variations of the R&D process impact the quantitative findings, they do not change the basic point of this paper: that the demographic transition and the ageing of the population improves output per capita under endogenous technological change.

8 Conclusion

The model in this paper allows the population structure to affect output per capita via three main channels: through the fraction of people who work, through capital accumulation, and through technological progress. This framework stands in stark contrast to standard macroeconomic life-cycle models, in which technological change is exogenous. It also contrasts with most models of endogenous growth, which hide the entire population structure in a representative household. A key point throughout the paper is that this matters for how we think about demographic change and its impact on output per capita.

My main findings suggest that current and projected US demographic change from 1950 onward raises output per capita and that this effect is quantitatively large; at least on par with the growth contribution from US educational attainment over the second half of the twentieth century. I show that this is primarily due to the inclusion of endogenous technological change. Removing this channel completely reverses the positive impact. Overall, these findings challenge a seemingly conventional wisdom that current demographic developments are detrimental to economic activity.

The framework employed here admittedly leaves out several potentially important channels, such as human capital accumulation, automation, or international technology flows. However, papers that consider endogenous responses in human capital or automation typically find that these extra adjustment margins improve the impact of population ageing on output per capita, much like the inclusion of technological progress in this paper. Moreover, the demographic transition is a global phenomenon, so imported technologies ultimately stem from countries that follow similar population trends. Thus, if anything, I would expect these mechanisms to only strengthen the main takeaways of the paper.

The results also raise questions about the conclusions drawn in other literatures that rely on exogenous growth models. For example, the pension system is a key determinant of the savings rate and, since the savings rate is central to technical change here, an immediate question is therefore whether social security reform becomes more or less costly under R&D-driven growth. Other topics include migration policy, different types of fiscal policy reform, and optimal taxation. Jones (2022b), for instance, suggests that the optimal income tax progressivity is significantly altered by the endogeneity of technological change. The model considered here could serve as a basis to analyse these questions further.

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Endogenous Technological Change Along the Demographic Transition Online Supplement

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31 July 2025

This supplement provides (a) derivations of the balanced growth rates in the main model; (b) details on the estimation of the life-cycle productivity profile; (c) details on the calibration of taxes; (d) details on the internally calibrated parameters; (e) additional sensitivity analyses; and (f) additional figures.

Appendix A Deriving the Stationary Growth Rates

The stationary growth rates of the benchmark model and of the version with labour as the only R&D input are straightforwardly derived following Jones (1995) and Boppart and Krusell (2020).

A.1 TFP, Output Per Capita, Hours Per Worker

In a stationary equilibrium, interior solutions to the household problem are characterised by the Euler equation, the intratemporal first-order condition for hours worked, and the budget constraint:

$$\frac{\Omega_{jc}}{1+\tau^c} c_j^{-\sigma} = \Big(1 + r(1-\tau^k)\Big) \left\{ (1-s_j)\,\Omega_a \left(a_{j+1}^b\right)^{-\sigma} + s_j\beta \frac{\Omega_{j+1,c}}{1+\tau^c} \mathbb{E}\Big[c_{j+1}^{-\sigma} \,\Big|\, \eta \Big] \right\}, \tag{A.1}$$

$$\psi h_j^{1/\theta} = \Omega_{jc} \frac{(1-\tau^{wm}(w\ell_j) - \tau^b) w \varepsilon_j \eta}{c_j^{\sigma}(1+\tau^c)}, \tag{A.2}$$

$$a_{j+1} + (1+\tau^c)c_j = \big(1 + r(1-\tau^k)\big)a_j + \big(1-\tau^w(w\ell_j) - \tau^b\big)w\ell_j + tr + b(R_j), \tag{A.3}$$

where $\tau^{wm}(e) \equiv \frac{\partial}{\partial e}\big(\tau^w(e)e\big)$ in (A.2) denotes the marginal income tax rate. Let g_x denote the stationary growth rate of a variable x. By the Euler equation, constant growth of consumption requires a constant interest rate. Wages are standard neoclassical, so in a stationary equilibrium with a constant interest rate and capital-output ratio, wages grow with the rate of TFP. For the first-order condition (A.2) to hold along a balanced growth path, we then necessarily need $(1+g_h)^{1/\theta} = (1+g_Z)(1+g_c)^{-\sigma}$. Likewise, the budget constraint (A.3) is only consistent with balanced growth if consumption grows with the rate of output per capita and labour income: $1+g_c = 1+g_y = (1+g_Z)(1+g_h)$. Meanwhile, TFP is just the measure of intermediate firms raised to a power, $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$, so its growth rate is similarly the growth rate of

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intermediate firms raised to the same power. Combining these conditions yields that the growth rates of TFP, output per capita, and hours per worker must satisfy

$$1+g_Z = (1+g_z)^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}, \qquad 1+g_y = (1+g_Z)^{\frac{1+\theta}{1+\theta\sigma}}, \qquad 1+g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}, \tag{A.4}$$

and substituting $1+g_z = (1+n)^\chi$ for some χ into Equation (A.4) gives the growth rates in Definition 2.

A.2 Intermediate-Firm Growth Rate: Baseline Model

Next, the intermediate-firm growth rate is given by $1+g_z = 1-\delta_z+\nu Q^\lambda z^{\phi-1}$, and this is constant if and only if the last term on the right-hand side is constant. The latter holds only if

$$(1+g_z)^{1-\phi} = (1+g_Q)^\lambda. \tag{A.5}$$

The growth rate of R&D investment equals the aggregate output growth rate $(1+g_Z)(1+g_L)$ by the goods market condition. Employment grows by the rate of the population $1+n$ whereas labour productivity is constant for a fixed population age structure. The labour force growth rate is therefore given by $1+g_L = (1+g_h)(1+n)$. Together with Equation (A.4), this allows us to rewrite the growth rate of R&D investment into

$$1+g_Q = (1+g_Z)(1+g_h)(1+n) = (1+g_z)^{\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(1+n).$$

Plugging this into (A.5) and rearranging terms yields the growth rate in Definition 2 in the paper:

$$1+g_z = (1+n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1-\phi-\lambda\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}. \tag{A.6}$$

A.3 Intermediate-Firm Growth Rate: R&D With Only Labour

If R&D uses only labour, then the R&D process is given by $1+g_z = 1-\delta_z+\nu L_z^\lambda z^{\phi-1}$. In steady state, R&D labour L_z must grow by the rate of total labour supply according to the labour market condition. Again inspecting the right-hand side, we thus get a constant growth rate $1+g_z$ if and only if

$$(1+g_z)^{1-\phi} = (1+g_L)^\lambda. \tag{A.7}$$

Using Equation (A.4), we can rewrite the labour force growth rate into

$$1+g_L = (1+g_h)(1+n) = (1+g_z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(1+n).$$

Plugging this into (A.7) and rearranging terms yields the growth rate

$$1+g_z = (1+n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1-\phi-\lambda\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}. \tag{A.8}$$

Note that TFP growth collapses to the benchmark growth rate in Jones (1995) if we consider a steady state with constant hours worked (via log preferences, $\sigma \to 1$) and a substitution parameter for intermediates that exactly equals the capital share parameter ($\rho = \alpha$).

Appendix B Estimating the Life-Cycle Earnings Profile

I parametrise the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ as the exponential of a quadratic age polynomial: $\varepsilon_j = \exp\left\{\vartheta_0 + \vartheta_1 j + \vartheta_2 j^2\right\}$. In the model, the hourly wage of an individual i of age j at time t is given by $w_{ijt} = w_t \varepsilon_j \eta_{ijt}$, where w_t is a common wage trend and η_{ijt} captures any idiosyncratic differences. This motivates the fixed effects regression

$$\ln w_{ijt} = \varrho_t + \varrho_i + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2 + u_{ijt}, \tag{B.1}$$

where ϱ_t is a time fixed effect, ϱ_i is an individual fixed effect, and u_{ijt} is an error term. Equation (B.1) implicitly captures cohort effects through the individual fixed effects and it is well known that collinearity between age, time, and cohorts prohibits simultaneous identification of these effects. As a partial remedy, I use the approach advocated by Heckman and Robb (1985) and replace the time fixed effect by two macroeconomic variables which plausibly proxy for the underlying unobserved time variables in the context of an earnings regression: log of the aggregate real wage level and the percentage point deviation of the unemployment rate from its long-run mean. The former corresponds to $\ln w_t$ and controls for secular wage growth and the latter (which is also used by French, 2005) controls for fluctuations in the business cycle.

I estimate Equation (B.1) with micro data on earnings from the nationally representative SRC sample of the Panel Study of Income Dynamics (PSID) for survey years 1968 to 2019 (which correspond to calendar years 1967 to 2018). Individual wages are imputed as total annual labour income divided by annual hours worked. The aggregate wage used to proxy the time fixed effect is obtained from the national accounts by dividing total private industry wages (BEA NIPA Table 2.3) by total private industry hours worked (BEA NIPA Table 6.9). The unemployment rate is taken from the Bureau of Labor Statistics (BLS, series ID LNS14000000). All nominal variables are deflated into 2012 dollars using the PCE price index (BEA NIPA Table 2.3.4).

For the benchmark estimation, I impose standard sample restrictions (see for instance French, 2005, Heathcote, Storesletten and Violante, 2010, and Huggett, Ventura and Yaron, 2011): I select male household heads with no inconsistencies in reported age, who work between 300 and 5,840 hours a year (30 percent of part time and twice full time, respectively), and whose hourly wage exceeds \$3 per hour and does not exceed \$100 per hour in 2012 dollars. I consider individuals between the ages of 18 and 75. This goes against the standard practice of excluding ages at the beginning and end of the working life to avoid sample selection issues relating to work-life entry and exit. This choice is motivated by the need for an efficiency profile for all ages above 20, given that retirement in the model is endogenous. The alternative, estimating the age profile on individuals between, say, the ages of 25 and 60, instead requires extrapolation of the age profile to younger and older ages, and it is not clear that this approach is preferable. An upper bound at 75 is nevertheless imposed to ensure there are at least 100 observations in each age group. Extrapolation beyond 75 is inconsequential, since between 95 and 99 percent of model households retire before 75. The final sample consists of 90,832 person-year observations.

Table B.1 shows the estimation results along with several robustness checks. Column (1) corresponds to the age profile in Figure F.5a in Appendix F. Column (2) shows standard OLS estimates and column (3) includes only individual fixed effects. In both cases, secular wage growth is interpreted as life-cycle earnings differences, and this generates steeper profiles; productivity at peak age is 115 to 130 percent larger than the initial age, compared to 70 percent for the main estimation. This underlines the importance

Table B.1. Estimation of deterministic age-efficiency profile.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	−1.3734***	0.9857***	1.1419***	−0.9102***	−1.2920***	−1.0902***	−1.1382***
	(0.2250)	(0.0191)	(0.0365)	(0.1783)	(0.2043)	(0.1655)	(0.1045)
Age	0.0734***	0.0954***	0.0835***	0.0813***	0.0709***	0.0610***	0.0606***
	(0.0020)	(0.0010)	(0.0018)	(0.0018)	(0.0018)	(0.0015)	(0.0010)
Age squared	−0.0008***	−0.0010***	−0.0008***	−0.0008***	−0.0008***	−0.0007***	−0.0007***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.160	0.121	0.156	0.162	0.158	0.152	0.153

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours. Regressors of interest: quadratic age polynomial. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

of controlling for time effects. Column (4) changes the aggregate wage measure to average hourly earnings of production and nonsupervisory employees (BLS, series ID CES0500000008). This wage exhibits lower growth in recent decades than the imputed wage from the BEA, and the estimation results are therefore similar to columns (2) and (3).[1] Columns (5) to (7) expand the sample to include spouses, partners, and female household heads and add additional individual-level controls that may change over time. These additions lower the point estimates somewhat, although these samples raise additional concerns for sample selection and also provide worse fits to the data as measured by the adjusted R^2.

Lastly, for the model scenario in which wages are an increasing function of hours worked in Appendix E, I run identical estimations to above with the only exception that individual wages are constructed as (total annual labour income)/(annual hours worked)$^{1.415}$ in the PSID data. Table B.2 shows the results from these regressions.

Appendix C Constructing Tax Rates

This section explains the calibration of the model taxes. First, I describe the construction of the aggregate tax rates from the national accounts, then the estimation a progressive income tax function, and finally the calibration of the income tax function used in the model (which builds on the former two).

[1] I use the BEA wage measure as the benchmark since the BLS wage is more limited in scope.

Table B.2. Estimation of deterministic age-efficiency profile with part-time wage penalty.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	-4.1083^{***}	-1.8276^{***}	-1.6727^{***}	-3.6135^{***}	-4.0415^{***}	-3.9382^{***}	-3.9195^{***}
	(0.2291)	(0.0200)	(0.0366)	(0.1809)	(0.2084)	(0.1669)	(0.1121)
Age	0.0540^{***}	0.0757^{***}	0.0634^{***}	0.0616^{***}	0.0515^{***}	0.0438^{***}	0.0400^{***}
	(0.0020)	(0.0010)	(0.0018)	(0.0018)	(0.0018)	(0.0014)	(0.0010)
Age squared	-0.0006^{***}	-0.0008^{***}	-0.0006^{***}	-0.0006^{***}	-0.0006^{***}	-0.0005^{***}	-0.0004^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.123	0.094	0.120	0.125	0.120	0.110	0.113

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours$^{1.415}$. Regressors of interest: quadratic age polynomial. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

C.1 Aggregate Tax Rates

The methodology to construct the aggregate tax rates on consumption, capital, and labour income is taken off-the-shelf from Fernández-Villaverde *et al.* (2015), which in turn builds on Jones (2002) and Mendoza, Razin and Tesar (1994). In short, each tax rate is given by aggregating all relevant tax revenues at the general government level and then dividing by the corresponding tax base. All data for this exercise are taken from BEA NIPA tables. Table C.1 summarises the variables that I use.

The average consumption tax rate τ^c is given by

$$\tau^c = \frac{TPI - PRT}{C - (TPI - PRT)}. \tag{C.1}$$

The numerator of (C.1) is the revenue from consumption taxation. I subtract property taxes from total taxes on production because homeowners in the national accounts are treated as businesses that rent their properties to themselves. Property taxes are therefore incorporated as taxes on capital instead. The consumption tax base in the denominator is total personal consumption expenditures net of consumption taxes paid (that is, the pre-tax value of consumption).

Table C.1. Tax variables in the national accounts.

Variable	Explanation	Source	
C	Personal consumption expenditures	BEA NIPA Table 1.1.5	line 2
EC	Compensation of employees	BEA NIPA Table 1.12	line 2
W	Wages and salaries	BEA NIPA Table 1.12	line 3
PRI	Proprietors' income[a]	BEA NIPA Table 1.12	line 9
RI	Rental income of persons[a]	BEA NIPA Table 1.12	line 12
CP	Corporate profits[a]	BEA NIPA Table 1.12	line 13
NI	Net interest and miscellaneous payments	BEA NIPA Table 1.12	line 18
PCT	Personal current taxes	BEA NIPA Table 3.1	line 3
TPI	Taxes on production and imports	BEA NIPA Table 3.1	line 4
CT	Taxes on corporate income	BEA NIPA Table 3.1	line 5
CSI	Contributions for government social insurance	BEA NIPA Table 3.1	line 7
PRT	Property taxes	BEA NIPA Table 3.3	line 9
$LABSH$	Labour share of national income	BEA NIPA Table 1.12[b]	

[a] With inventory valuation adjustment and capital consumption adjustment.
[b] See Appendix D for details.

The NIPA tables do not provide a breakdown of personal current taxes into labour and capital income. To make this split, I construct an average personal income tax rate τ^p as an intermediate step via

$$\tau^p = \frac{PCT}{W + LABSH \cdot PRI + CI}, \qquad \text{where} \qquad CI \equiv (1 - LABSH)PRI + RI + CP + NI.$$

The numerator is the sum of personal current taxes at the federal, state and local levels. The tax base is the sum of wages, proprietors' income, and capital income (CI). Here, proprietors' income is assumed to be split between labour and capital income in the same proportion as overall national income, which is consistent with the construction of the labour share in Appendix D.

I then estimate the total revenue from personal taxes on income and capital as $\tau^p(W + LABSH \cdot PRI)$ and $\tau^p CI$, respectively. The average labour income and capital tax rates are subsequently given by

$$\tau^w = \tau^p \frac{W + LABSH \cdot PRI}{EC + LABSH \cdot PRI} \qquad \text{and} \qquad \tau^k = \frac{\tau^p CI + CT + PRT}{CI + PRT}. \tag{C.2}$$

While not used directly in the benchmark model, one may also construct a social security tax rate as

$$\tau^b = \frac{CSI}{EC + LABSH \cdot PRI}. \tag{C.3}$$

The sum of τ^w and τ^b gives the average labour income tax rate used by Fernández-Villaverde *et al.* (2015). Figure C.1 plots the estimated tax rates, which highlights that imposing constant tax rates in the model for consumption, labour income and capital is a reasonable assumption.

C.2 Estimating the Income Tax Rate Function

To estimate the income tax function, I compute average tax rates at hypothetical levels of income and fit Equation (35) in the main paper to these synthetic data. This process follows the methodology of the

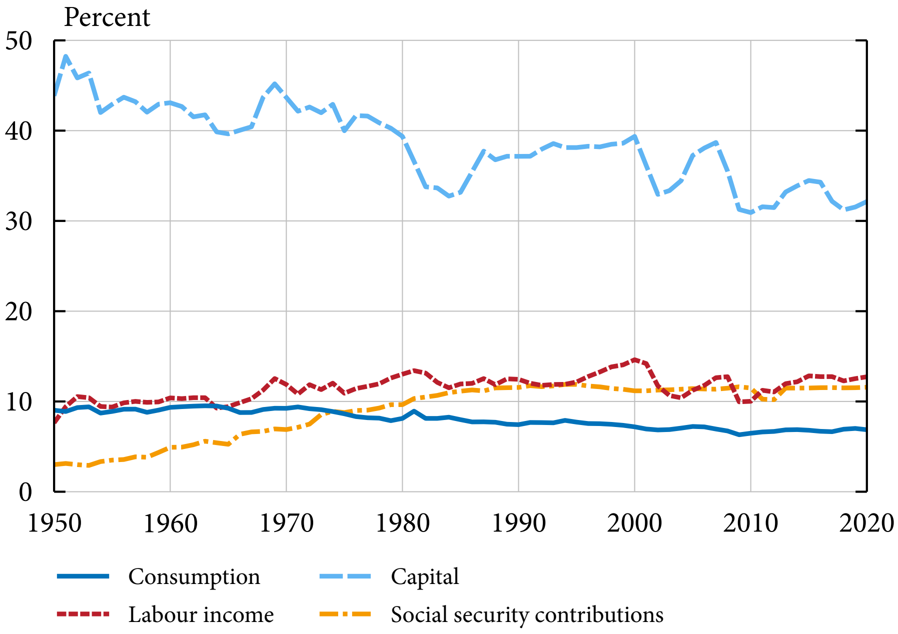

Figure C.1. Aggregate tax rate estimates from the national accounts.

OECD tax database for the United States, which creates effective tax rates by applying applicable tax rules and regulations for given years and earnings levels and then dividing the resulting net tax liabilities by gross earnings. These calculations include taxation at all levels of government for a household assumed to live in Detroit, Michigan.

For simplicity, I only consider single households without children, in line with the primary estimates published by the OECD.[2] This choice is of secondary importance since I eventually scale the estimated tax function to match the national accounts. I also abstract from social security contributions since those are modelled separately in my framework. The subsections below outline the formulas and parameters for this particular case.[3]

C.2.1 Taxable Earnings

Taxable earnings at government level $\mathtt{x} \in \{\mathtt{fed}, \mathtt{state}, \mathtt{local}\}$ is given by gross income GI minus a tax allowance $\text{TAXALLOW}_{\mathtt{x}}$, provided that this is positive:

$$e^{\mathtt{x}}(\text{GI}) = \max\{\text{GI} - \text{TAXALLOW}_{\mathtt{x}}, 0\}.$$

At the federal level, the allowance consists of a standard deduction STDALLOW and a personal exemption $\text{EXEMPT}_{\text{fed}}$. The personal exemption is reduced at a taper rate φ^T_{ex} for every USD 2,500 that gross income exceeds the threshold THOLD_{ex}. At the state and local levels, the allowances are fixed personal exemptions $\text{EXEMPT}_{\text{state}}$ and $\text{EXEMPT}_{\text{local}}$, respectively. Thus,

$$\text{TAXALLOW}_{\text{fed}} = \text{STDALLOW} + \text{EXEMPT}_{\text{fed}}\left(1 - \varphi^T_{\text{ex}}\left\lceil\frac{\max\{\text{GI} - \text{THOLD}_{\text{ex}}, 0\}}{2500}\right\rceil\right),$$

[2] See Table I.5, available for download at OECD.Stat.

[3] The implementation code (available upon request) also incorporates different household compositions (with respect to children and marital status) and social security contributions. Supplementary documentation for these cases and the parameter values used are given in the OECD publication *Taxing Wages* available at the OECD iLibrary.

$$\text{TAXALLOW}_{\text{state}} = \text{EXEMPT}_{\text{state}},$$

$$\text{TAXALLOW}_{\text{local}} = \text{EXEMPT}_{\text{local}},$$

where $\lceil\cdot\rceil$ is the ceiling function: $\lceil x\rceil = \min\{n \in \mathbb{Z} : n \geq x\}$.

C.2.2 Taxes

Taxable earnings at the state and local levels are subject to flat tax rates τ^{state} and τ^{local}. The federal income tax is progressive, with higher marginal tax rates at higher levels of income. Consider N federal tax brackets with marginal tax rates $\tau_1^{\text{fed}}, \ldots, \tau_N^{\text{fed}}$ starting at earnings thresholds $\bar{e}_1, \ldots, \bar{e}_N$, where $\bar{e}_1 = 0$. Given a largest applicable federal tax bracket $I = \max\{i : e^{\text{fed}}(\text{GI}) > \bar{e}_i\}$, the tax liability at each level of government is then given by functions $T^{\text{x}}(\text{GI})$ of gross income as follows:

$$T^{\text{fed}}(\text{GI}) = \sum_{i=1}^{I-1} \tau_i^{\text{fed}} \left(\bar{e}_{i+1} - \bar{e}_i\right) + \tau_I^{\text{fed}} \left(e^{\text{fed}}(\text{GI}) - \bar{e}_I\right),$$

$$T^{\text{state}}(\text{GI}) = \tau^{\text{state}}\, e^{\text{state}}(\text{GI}),$$

$$T^{\text{local}}(\text{GI}) = \tau^{\text{local}}\, e^{\text{local}}(\text{GI}).$$

C.2.3 Tax Credits

The OECD considers two types of federal tax credits for households without children: the Earned Income Tax Credit (EIC) and the Making Work Pay tax credit (MWP). The EIC and the MWP provide refundable tax credits equal to some fractions φ_{eic} and φ_{mwp} of gross income up to some maximum amounts $\varphi_{\text{eic}}\,\overline{\text{eic}}$ and $\overline{\text{mwp}}$. The tax credits are phased out at taper rates φ^T_{eic} and φ^T_{mwp} once gross income exceeds thresholds $\text{THOLD}_{\text{eic}}$ and $\text{THOLD}_{\text{mwp}}$. The total tax credit amounts from these programs are thus given by

$$\text{eic}(\text{GI}) = \max\left\{\varphi_{\text{eic}} \min\left\{\text{GI}, \overline{\text{eic}}\right\} - \varphi^T_{\text{eic}} \max\left\{\text{GI} - \text{THOLD}_{\text{eic}}, 0\right\}, 0\right\}$$

and

$$\text{mwp}(\text{GI}) = \max\left\{\min\left\{\varphi_{\text{mwp}}\text{GI}, \overline{\text{mwp}}\right\} - \varphi^T_{\text{mwp}} \max\left\{\text{GI} - \text{THOLD}_{\text{mwp}}, 0\right\}, 0\right\}.$$

Total federal tax credits is the sum of EIC and MWP. At the state level, the OECD includes the Michigan Earned Income Tax Credit, which is an additional refundable credit equal to a fraction φ_{meic} of the federal EIC amount. The local level incorporates the Michigan City Income Tax Credit (CTC) which is a nonrefundable credit equal to some fraction of the total local tax liability $T^{\text{local}}(\text{GI})$ up to some maximum amount $\overline{\text{ctc}}$. Below this upper bound, the CTC credit rates decline with income. Consider N credit rate brackets with marginal credit rates $\varphi_{1,\text{ctc}}, \ldots, \varphi_{N,\text{ctc}}$ starting at tax liability thresholds $\overline{T}_1, \ldots, \overline{T}_N$, where $\overline{T}_1 = 0$. Given a largest applicable tax credit bracket $I = \max\{i : T^{\text{local}}(\text{GI}) > \overline{T}_i\}$, the total tax credit at each level of government is then given by functions $C^{\text{x}}(\text{GI})$ of gross income as follows:

$$C^{\text{fed}}(\text{GI}) = \text{eic}(\text{GI}) + \text{mwp}(\text{GI}),$$

$$C^{\text{state}}(\text{GI}) = \varphi_{\text{meic}}\,\text{eic}(\text{GI}),$$

$$C^{\text{local}}(\text{GI}) = \min\left\{\sum_{i=1}^{I-1}\varphi_{i,\text{ctc}}\left(\overline{T}_{i+1}-\overline{T}_i\right) + \varphi_{I,\text{ctc}}\left(T^{\text{local}}(\text{GI})-\overline{T}_I\right),\ \overline{\text{ctc}}\right\}.$$

C.2.4 Effective Income Tax Rate

The effective income tax rate $\tau^w(\text{GI})$ at gross income GI is the total tax liability net of tax credits measured as a percentage of gross income:

$$\tau^w(\text{GI}) = \frac{1}{\text{GI}}\sum_{\text{x}\in X}\left(T^{\text{x}}(\text{GI}) - C^{\text{x}}(\text{GI})\right),$$

where $X = \{\texttt{fed}, \texttt{state}, \texttt{local}\}$. In the practical implementation of these calculations, we consider an average gross income level $\overline{\text{GI}}$ and then express all other gross incomes as a percentage of that average.

C.2.5 Estimation

Using the methodology above, I create effective income tax rates on a grid of gross incomes for each year between 2000 and 2022. The grid is linearly spaced with 401 points, ranging from 0 to a multiple 20 of average gross income. The parameter values for this exercise are collected from the OECD and are available upon request. I then fit the income tax function $\tau^w(\text{GI}) = \kappa_0\left[1-\left(\kappa_2\left(\frac{\text{GI}}{\overline{\text{GI}}}\right)^{\kappa_1}+1\right)^{-\frac{1}{\kappa_1}}\right]+\kappa_3$ by a nonlinear OLS to these tax rates.

Figure C.2 shows the constructed tax rates for the lower end of the income grid together with the corresponding fit and its estimated coefficients. Even though this period saw two major tax reforms (the Economic Growth and Tax Reconciliation Relief Act of 2001 and the Tax Cuts and Jobs Act of 2017) and underwent three economic downturns (the early 2000s recession, the Great Recession, and the COVID-19 recession), effective income tax rates remain stable over time. Therefore, the estimated tax function provides a close fit, as seen by the high R^2 of 0.97.

C.3 Changing the Tax Rate Level While Maintaining Progressivity

Once the income tax function is estimated, I adjust its level so that the tax rate at average earnings matches the tax rate from the national accounts. To this end, I follow Guvenen, Kuruscu and Ozkan (2014) to ensure that the degree of progressivity remains the same before and after. Thus, let $\widetilde{\tau}(e)$ be some tax rate function of the Gouveia and Strauss (1994) form:

$$\widetilde{\tau}(e) = \widetilde{\kappa}_0\left[1-\left(\widetilde{\kappa}_2\left(\frac{e}{\overline{e}}\right)^{\widetilde{\kappa}_1}+1\right)^{-\frac{1}{\widetilde{\kappa}_1}}\right]+\widetilde{\kappa}_3. \tag{C.4}$$

Denote its corresponding marginal tax rate by $\widetilde{\tau}^m(e) = \frac{\partial}{\partial e}\left(\widetilde{\tau}(e)e\right)$. To change the level of this tax function into a similar function $\tau(e)$ with parameters $\kappa_0, \ldots, \kappa_3$ without changing the progressivity, we need the ratio of net take-home shares at any two earnings levels e and e' to be the same in both tax systems:

$$\frac{1-\tau^m(e')}{1-\tau^m(e)} = \frac{1-\widetilde{\tau}^m(e')}{1-\widetilde{\tau}^m(e)}.$$

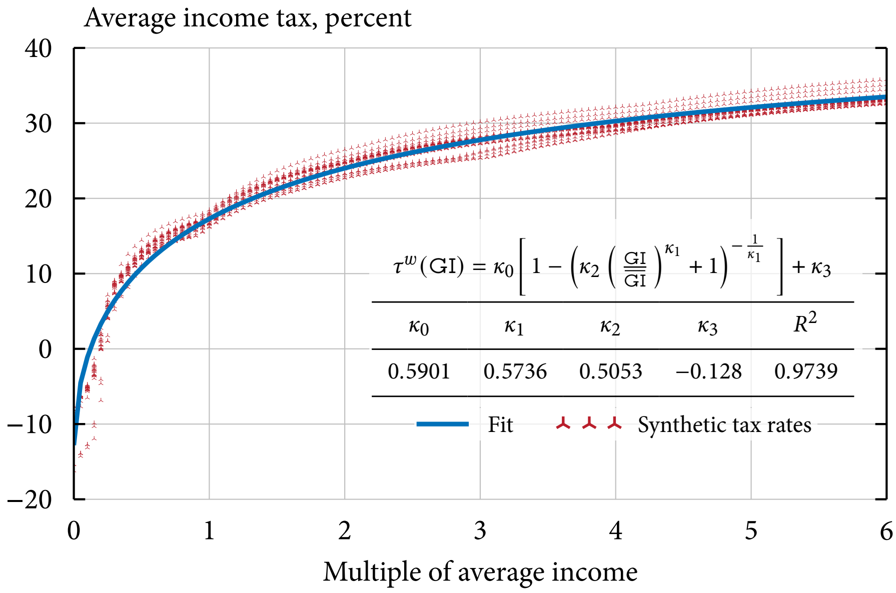

Figure C.2. Estimation of the income tax function.

This expression can be rearranged to obtain

$$\tau^m(e) = 1 - \bar{k}\big(1 - \tilde{\tau}^m(e)\big), \qquad \text{where} \qquad \bar{k} \equiv \frac{1 - \tau^m(e')}{1 - \tilde{\tau}^m(e')} \tag{C.5}$$

is a level ratio between the two tax systems that we are free to choose. Since $\tau(e)e = \int_0^e \tau^m(x)\,dx$, we can integrate Equation (C.5) to obtain an average tax rate of a similar form:

$$\tau(e) = 1 - \bar{k}\big(1 - \tilde{\tau}(e)\big). \tag{C.6}$$

Substituting Equation (C.4) into (C.6) and rearranging terms, we finally get

$$\tau(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa 1}} \right] + \kappa_3,$$

where $\kappa_0 \equiv \bar{k} \cdot \tilde{\kappa}_0$, $\kappa_1 \equiv \tilde{\kappa}_1$, $\kappa_2 \equiv \tilde{\kappa}_2$ and $\kappa_3 \equiv 1 - \bar{k}(1 - \tilde{\kappa}_3)$. The calibrated $\kappa_0, \ldots, \kappa_3$ in the benchmark model use the estimates in Figure C.2 as $\tilde{\kappa}_0, \ldots, \tilde{\kappa}_3$ and set the scale parameter $\bar{k}$ such that the tax rate at average income $\bar{e}$ matches the income tax rate τ^{NA} from the national accounts. Specifically, the latter requires that $\tau(\bar{e}) = \tau^{NA} = 1 - \bar{k}\big(1 - \tilde{\tau}(\bar{e})\big)$, which can be rearranged to give the scale parameter as

$$\bar{k} = \frac{1 - \tau^{NA}}{1 - \tilde{\tau}(\bar{e})}, \qquad \text{where} \qquad \tilde{\tau}(\bar{e}) = \tilde{\kappa}_0 \left[1 - \left(\tilde{\kappa}_2 + 1 \right)^{-\frac{1}{\tilde{\kappa}_1}} \right] + \tilde{\kappa}_3$$

is a function of estimated parameters only.

Appendix D Internal Calibration of Model Parameters

The internally calibrated parameters are obtained by a method of moments procedure using model-predicted and empirical data. Denote the vector of parameters by $\Theta = (\beta, \psi, \Omega_A, \delta_k, \delta_z, \alpha, \rho)$. Let y_t be the vector of empirical observations in period t for the variables whose moments we wish to match, and let $f_t(\Theta)$ be the corresponding model-predicted observations as a function of the parameters. The residual between model and data is then $u_t(\Theta) = y_t - f_t(\Theta)$ for $t = 1, \ldots, T$. I obtain the parameter vector from the method of moments estimator by setting the sample averages of $u_t(\Theta)$ to zero (up to some tolerance):

$$\widehat{\Theta} = \arg\min_{\Theta} \left(\frac{1}{T} \sum_{t=1}^{T} u_t(\Theta) \right)^{\top} \left(\frac{1}{T} \sum_{t=1}^{T} u_t(\Theta) \right).$$

The moment estimators for α and ρ have analytical solutions and can be directly determined from the data. The remaining parameters are estimated by simulation. The practical implementation of this estimation is the same as in Ludwig, Schelkle and Vogel (2012), which I refer to for further details. In short, the algorithm proceeds as follows:

(i) Start with the empirical moments as calibration targets.
(ii) Use these targets to calibrate the model out of steady state.
(iii) For the estimated parameters, compute the equilibrium transition path.
(iv) Obtain the corresponding model-predicted moments for the relevant transition periods.
(v) If the predicted moments exceed the empirical moments, decrease the steady-state calibration target (and vice versa).
(vi) Iterate until the predicted moments converge to the empirical counterparts.

Because the model begins before most official US data series, constructing the empirical moments requires some special care. The following summarises the data choices for the internal calibration.

- **Output:** I use Kendrick's (1961, Table A-III) real GDP series until 1929, inflate it with the implicit GNP deflator from his Tables A-IIa and A-IIb, and splice it with the BEA's GDP series (NIPA Table 1.1.5).
- **Capital stock:** I sum structures, producer durables, and nonfarm land from Goldsmith, Brady and Mendershausen's (1956) Table W-1 and splice it with BEA data on total fixed assets net of IPP assets (FAA Table 1.1).
- **Hours worked:** I splice Kendrick's (1961, Table A-X) aggregate hours estimates with Bureau of Labor Statistics data on hours for all workers in the total US economy. I then divide by the population aged 20 to 64, as reported in the Census Bureau's intercensal population tables, and convert to the unit interval assuming a 16-hour daily time endowment. The estimation considers normalized hours worked, $h/Z^{\theta(1-\sigma)/(1+\theta\sigma)}$, where Harrod-neutral TFP is obtained as the residual between gross output and labour and capital inputs in Kendrick's Table A-XIX using the labour shares below.
- **Life-cycle profile of assets in 1950:** I take this from the 1949 Survey of Consumer Finances, as reported in Goldsmith (1955, Table XLIII).
- **Depreciation of non-IPP capital:** before 1929, I use Goldsmith, Brady and Mendershausen's (1956, Table N-5) estimates and splice them with BEA data on total consumption of fixed capital less intellectual property products (NIPA Table 5.2.5).
- **Depreciation of IPP capital:** this does not exist before the NIPA start year, which restricts this moment to the 1929–1949 subperiod when it is available (NIPA Table 5.2.5).
- **Labour share:** I extend the BEA's national income data (NIPA Table 1.12) back to 1900 using income components from Martin (1939, Tables 4 and 41) and Goldsmith, Brady and Mendershausen (1956, Tables N-1 and N-5). The labour share is calculated as compensation of employees divided by GDP net of subsidies, taxes on production and imports, and proprietors' income. The latter subtraction

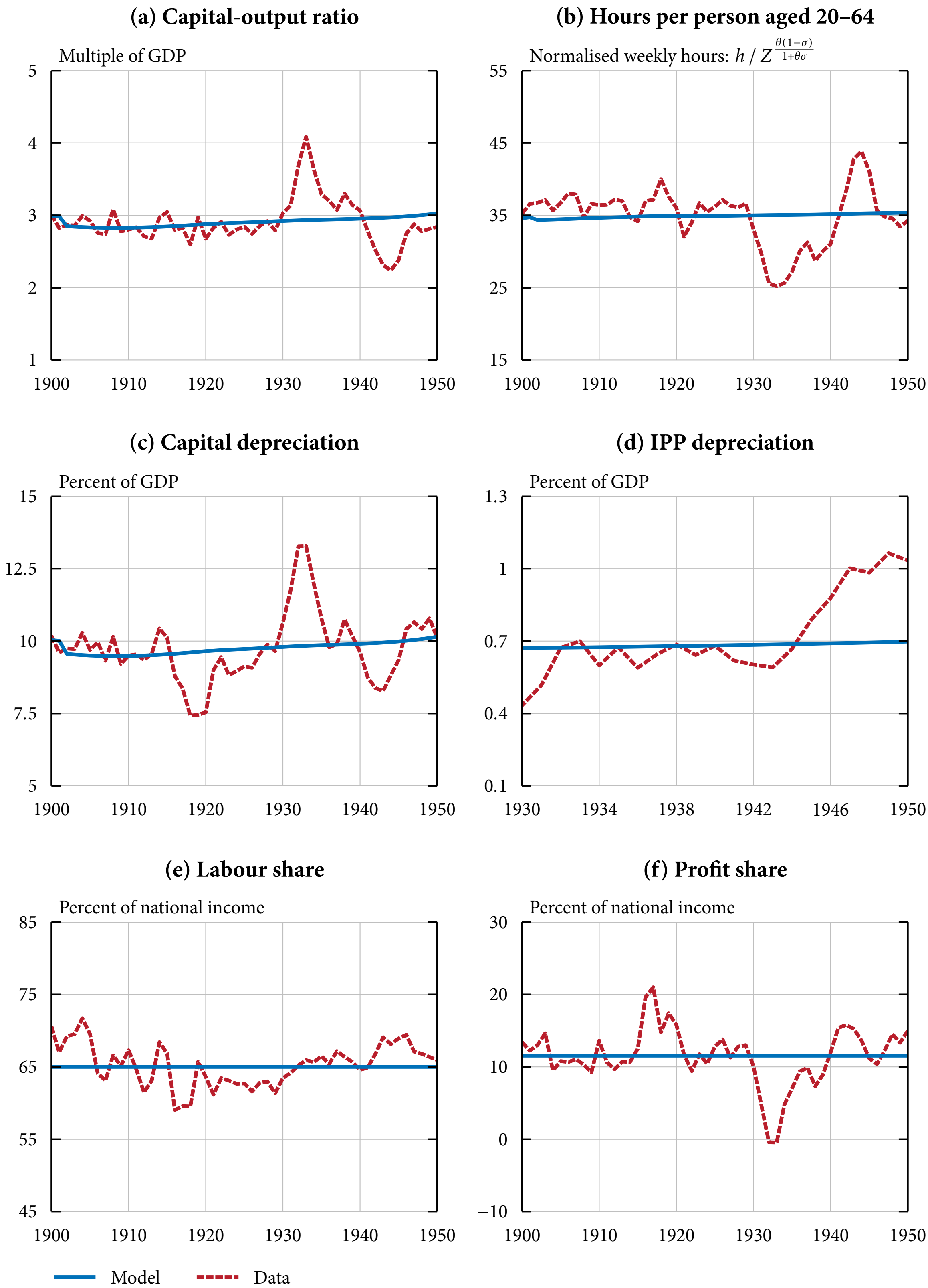

Figure D.1. Time series for simulated method of moments.

imposes the assumption that the self-employed use labour as intensively as the rest of the economy, per recommendation of Karabarbounis (2024).

- **Profit share:** is constructed analogously to the labour share using corporate profits—the sum of taxes on corporate income, net dividends, and undistributed corporate profits—as numerator, again assuming the same profit share for proprietors as in the corporate sector.

Figure D.1 plots the resulting empirical time series against the model's predictions (the life-cycle profile of assets is already shown in Figure 4a in the main body of the paper). The figure reveals that the model closely fits the data on average, with most deviations being due to the large swings associated with the Great Depression and its subsequent recovery.

Appendix E Additional Sensitivity Analyses

This appendix complements Section 7 in the main paper with additional robustness checks. Each robustness scenario recalibrates the parameters β, ψ, Ω_a, δ_k, and δ_z to match the baseline calibration targets if needed. The growth rate of each scenario is plotted against the baseline in Figure E.1, while Figure E.2 shows their growth decompositions. Overall, neither alternative alters any qualitative conclusions.

Increased retirement age. The social security system in the benchmark model has a fixed normal retirement age (NRA) and early/delayed pension scaling schedule, contrary to reality. Since the average age of retirement influences all growth mechanisms in the paper, I therefore consider an alternative which more accurately describes the NRA and the delayed retirement credits. Specifically, I increase the NRA to 66 for cohorts born between 1940 and 1956 and to 67 for subsequent cohorts. Moreover, the delayed retirement credit is increased by 0.5 percentage points for every other cohort between 1924 and 1943. That is, the delayed retirement credit is 3 percent for cohorts born before 1925, 3.5 percent for the 1925–1926 cohorts, . . . , 7.5 percent for the 1941–1942 cohorts, and 8 percent for all subsequent cohorts. Overall, these changes nevertheless leave the baseline results unaffected because most households in the baseline already retire between the ages of 65 and 70.

Deterministic earnings. In the baseline, households face uninsurable idiosyncratic productivity shocks that add a savings motive beyond the standard life-cycle motive. In an alternative, I remove this feature and consider deterministic earnings. Although this reduces inequality and the overall level of household savings, the difference to the baseline turns out to be negligible. The large one-off growth spike in this simulation is not a numerical error. Rather, this is a year in which households choose to start retiring one year later than prior cohorts. Since the model is deterministic, this change applies to everyone within a cohort, thus generating a large increase in the employment rate in this particular period.

No bequest motive. Without a bequest motive, an increase in life expectancy strengthens the life-cycle savings motive and unambiguously raises the saving rate. Warm-glow bequests act as a counteracting force, since higher survival rates dampen the savings motive of leaving bequests. To assess whether this is an important aspect, I set the utility weight of bequests (Ω_a) to zero and rerun the model. As expected, the growth impact is now larger, although it is quantitatively insignificant and changes no insights from the baseline.

Flat income tax. Rather than considering a progressive income tax, I analyse an alternative in which all households face a constant marginal (and average) income tax rate equal to that obtained from the national accounts: $\tau^w = 0.116$. Contrary to the deterministic earnings scenario, imposing a flat tax

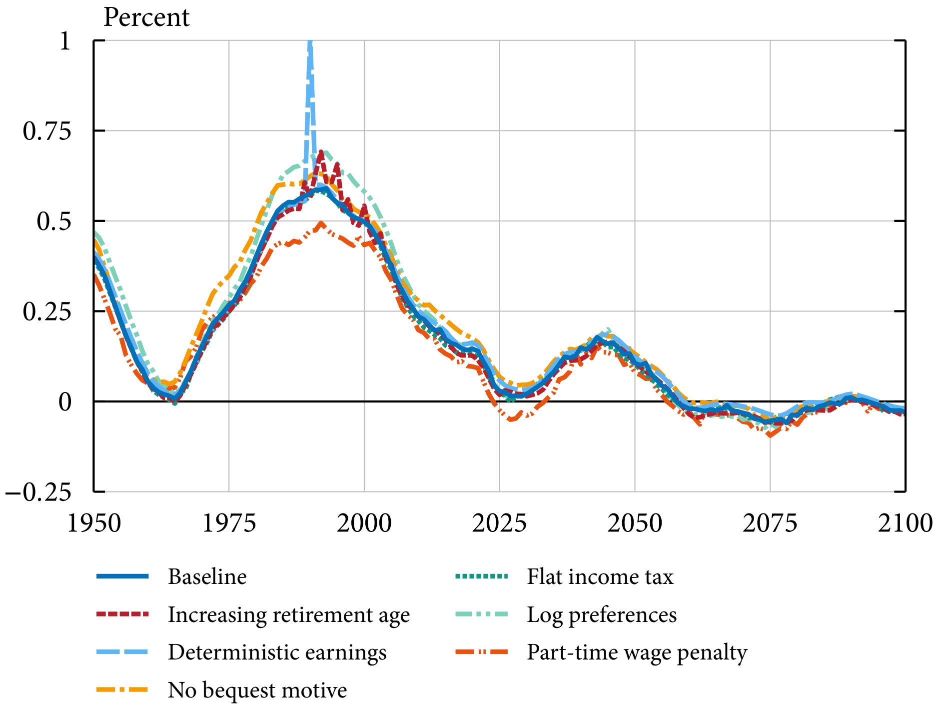

Figure E.1. Sensitivity check: growth rates with different model specifications.

increases inequality and the level of household savings. Again, the difference to the baseline is nevertheless negligible.

Log preferences. Another nonstandard feature is that I consider preferences of the Boppart and Krusell (2020) class that generate declining hours worked along a balanced growth path. By contrast, a large part of the macroeconomic literature restricts itself to the subset of this class defined by King, Plosser and Rebelo (1988), in which hours worked are constant in the long run. Constant long-run hours in my model are obtained as the special case when $\sigma \to 1$, so that flow utility becomes $u(c_j, h_j, a^b_{j+1}) = \Omega_{jc}\log(c_j) - \psi\frac{h_j^{1+1/\theta}}{1+1/\theta} + (1-s_j)\Omega_a \log(a^b_{j+1})$. In this case, the income and substitution effect on leisure exactly offset each other, so hours worked do not fall when growth is positive. Between 1950 and 2000, annual growth with logarithmic preferences is therefore about 0.1 percentage points higher. This difference is explained entirely by the different adjustments in hours worked. Growth during the twenty-first century does not change since, with wage growth around zero, the response in hours is similar in both scenarios.

Part-time wage penalty. Several authors stress the importance of nonconvexities in the budget set to generate endogenous retirement (see for instance Rogerson and Wallenius, 2013, and Ljungqvist and Sargent, 2014). Social security plays this role in the baseline model. Another commonly used nonconvexity is nonlinear wages, which is motivated by the empirical observation that part-time work does not pay as high hourly wage as full-time work. Thus, following French (2005), consider household labour earnings given by $w\varepsilon_j\eta h_j^{1+\xi}$, with $\xi \geq 0$. The labour market condition then changes to $L = \sum_j \int_X \varepsilon_j \eta h_j(x)^{1+\xi}\, \mathrm{d}\Phi_j$ and, by similar derivations as in Appendix A, the long-run growth rate of intermediate firms becomes

$$1 + g_z = (1+n)^{\chi}, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}.$$

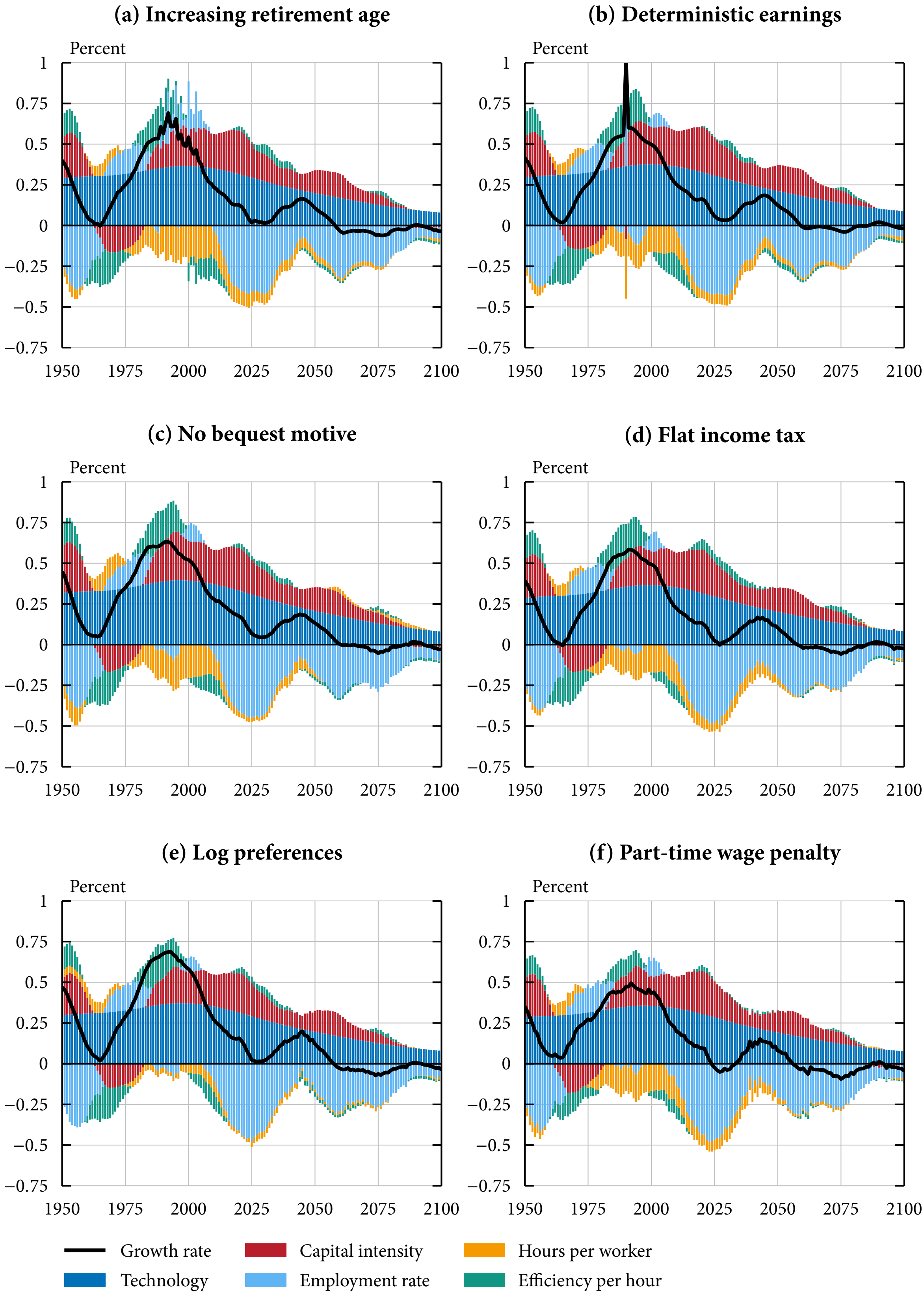

Figure E.2. Sensitivity check: growth decompositions with different model specifications.

The long-run growth rates of output per capita and hours per worker similarly become

$$1 + g_y = (1 + g_Z)^{\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}} \qquad \text{and} \qquad 1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma-\xi\theta(1-\sigma)}}.$$

For $\xi = 0$, wages are a linear function of hours worked and we obtain the benchmark model. Here, I follow French and set the value of ξ to 0.415 based on Aaronson and French's (2004) empirical finding that a 50 percent reduction in hours corresponds to a 25 percent lower hourly wage. I recalibrate the model under the assumption that $\xi = 0.415$ holds, which lowers the intertemporal elasticity of substitution, $\sigma = 1.84$, and flattens the age-efficiency profile.[4]

With these adjustments, the growth rate is similar to the baseline on average but exhibits more stable dynamics during the twentieth century. The latter is due to two changes to the average productivity per hour worked. First, declining hours worked negatively impacts average productivity since the productivity of an individual worker, $\varepsilon_j \eta h_j^{\xi}$, now includes hours worked. Second, the age-efficiency profile is flatter than in the baseline. Changes in the age composition of the labour force, which are more prominent in the twentieth century, therefore leads to smaller changes in average efficiency.

[4] For the baseline, I estimate the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ from a PSID wage measure obtained by dividing annual labour income with annual hours. Here, I assume that $\xi = 0.415$ holds and construct PSID wages as (total annual labour income)/(annual hours worked)$^{1.415}$ (see Appendix B for estimation details).

Appendix F Additional Figures

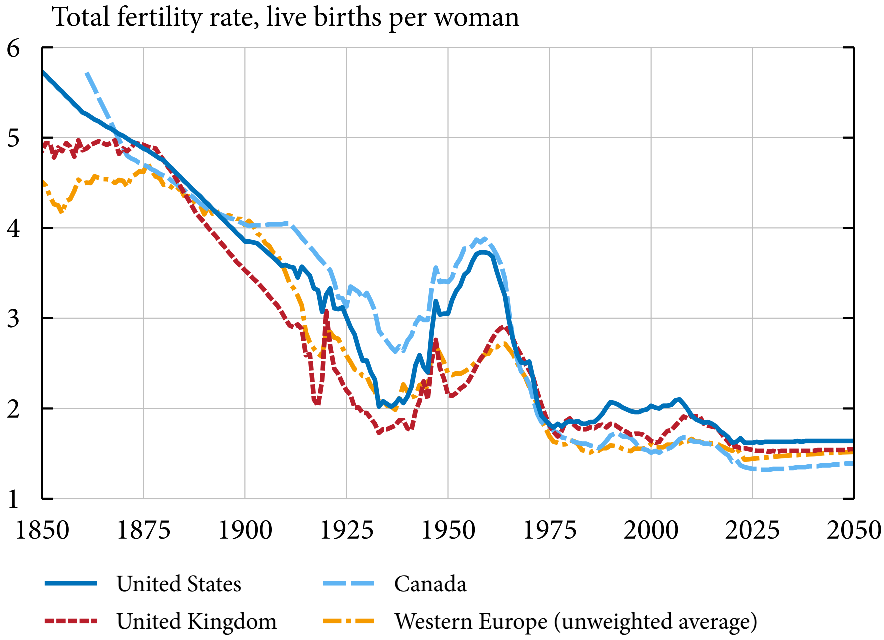

Figure F.1. Fertility across major economies.

Source. Gapminder Foundation (2024).

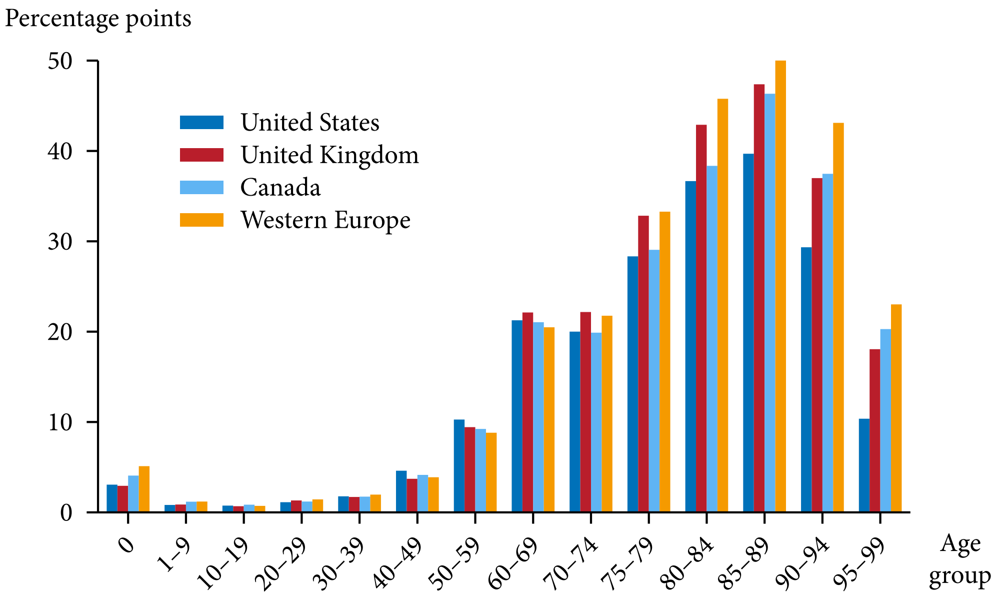

Figure F.2. Increase in survival probabilities by age group across major economies, 1950–2100.

Source. United Nations (2024).

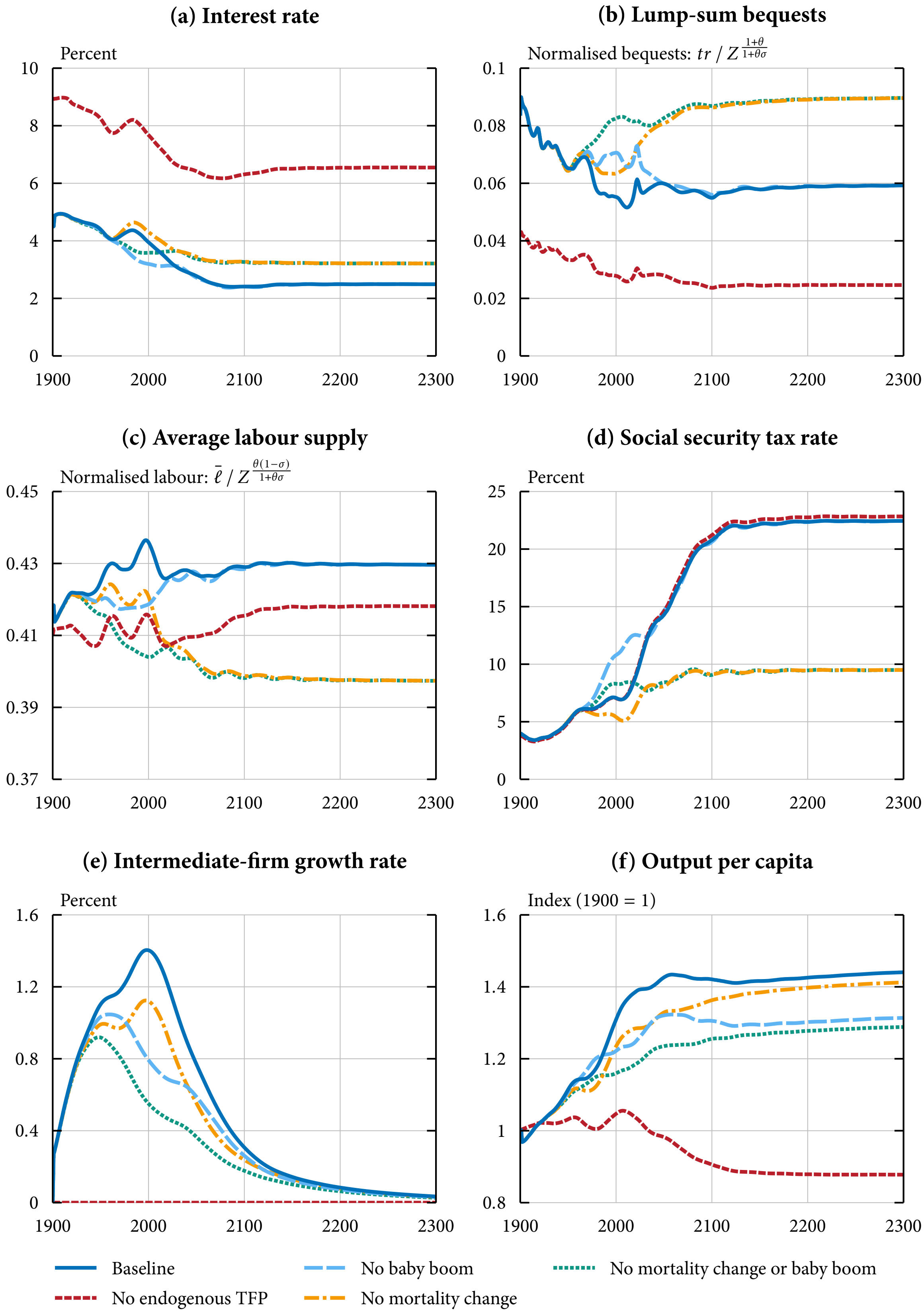

Figure F.3. Transition paths of equilibrium variables and output per capita.

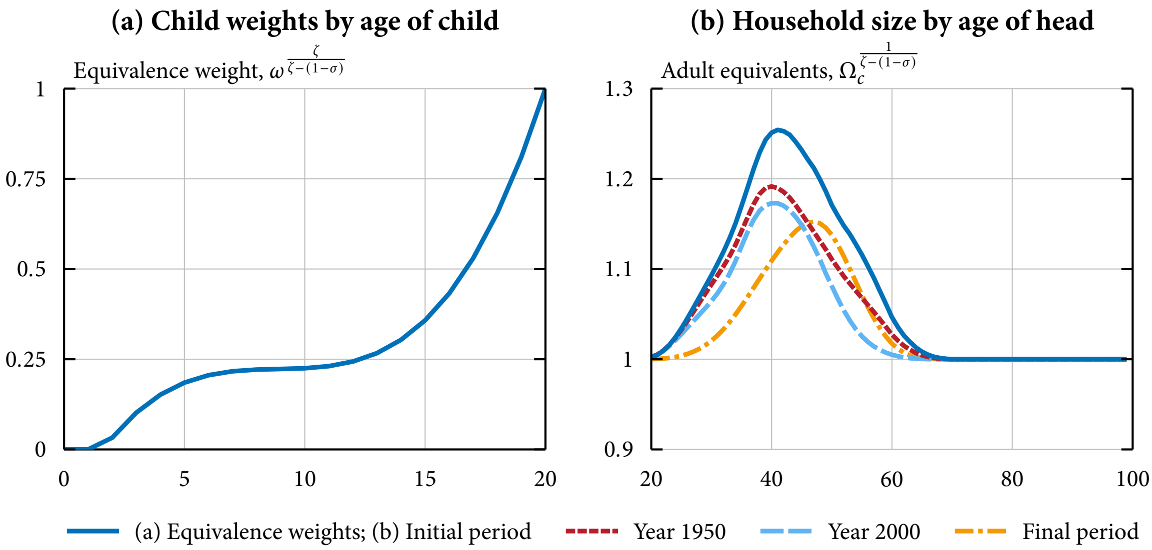

Figure F.4. Household size.

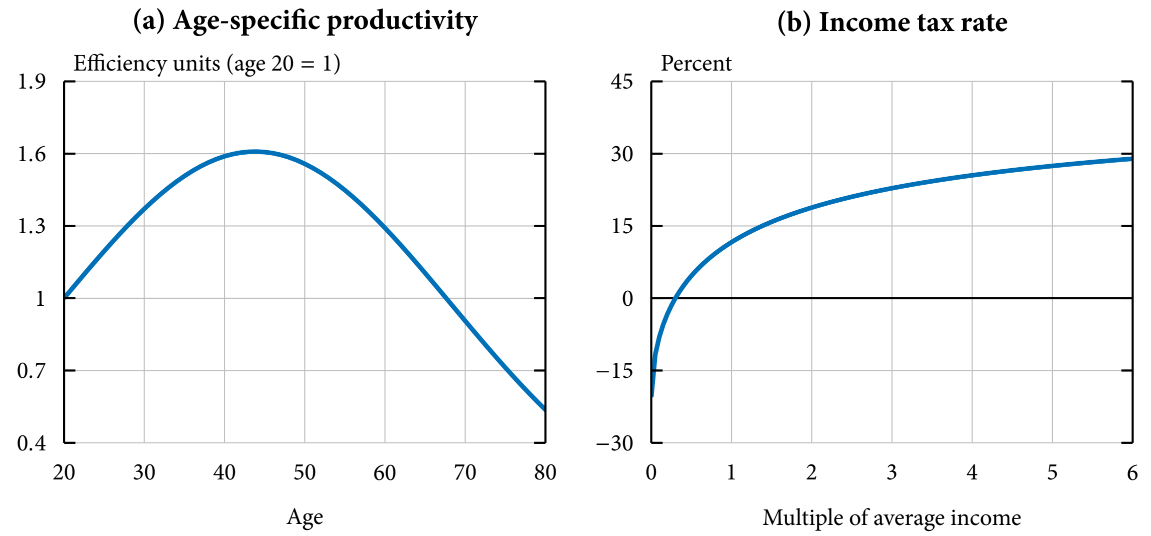

Figure F.5. Calibrated productivity and tax profiles.

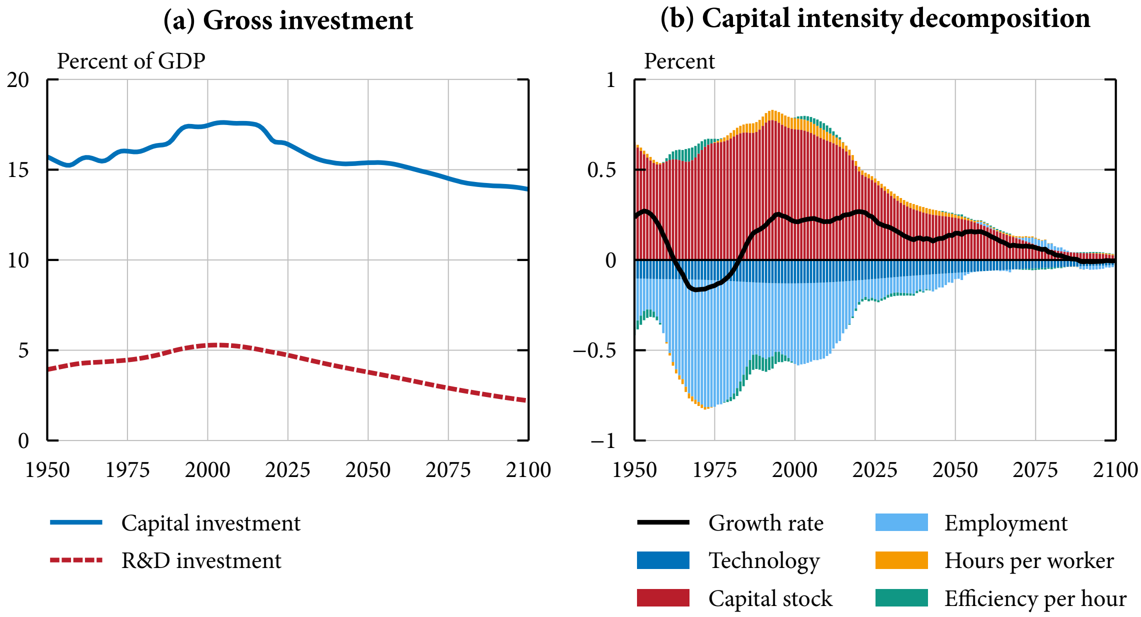

Figure F.6. Investment and capital deepening in the baseline scenario.

Notes. Figure F.6b displays the growth decomposition $\frac{\alpha}{1-\alpha} g_{K/Y} = \alpha(g_K - g_Z - g_E - g_h - g_\varepsilon)$, which is obtained by log differencing the capital intensity $\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{K}{ZL}\right)^{\alpha}$ together with the labour decomposition $L = E\,\bar{h}\,\bar{\varepsilon}$.

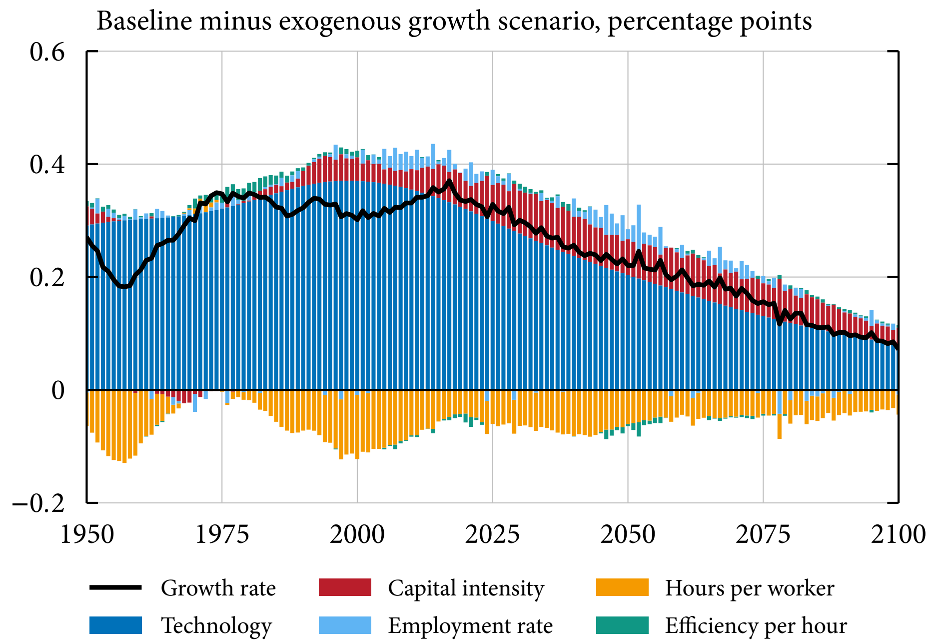

Figure F.7. Growth difference between the baseline and exogenous growth scenario.

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