A Nonhomothetic Price Index and Cost-of-Living Inequality
Online Supplement

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This supplement provides (i) proofs for the propositions and lemmas in the main text; (ii) an extension of the PIGL price index framework that allows for taste heterogeneity; (iii) a similar extension that admits hump-shaped expenditure shares; and (iv) additional tables and figures.

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Appendix A Proofs

A.1 Proof of Proposition 1

Proof. Inverting the indirect utility function in Equation (4) gives the expenditure function

$$c(u,\boldsymbol{p}) = \left[1 + \varepsilon\left(u + \frac{\nu}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}).$$

Suppose that the reference utility u corresponds to the observed consumption expenditure level e_s in a base period s, such that $u \equiv V(e_s, \boldsymbol{p}_s)$ and $c(u,\boldsymbol{p}_s) = e_s$. Substituting the period-s indirect utility function in Equation (4) into a period-t expenditure function and rearranging terms yields

$$\begin{aligned}
c(u,\boldsymbol{p}_t) &= e_s\left[1 + \frac{\varepsilon\nu}{\gamma}\left(\frac{B(\boldsymbol{p}_s)}{e_s}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p}_s)}{B(\boldsymbol{p}_s)}\right)^{\gamma}\left\{\left(\frac{D(\boldsymbol{p}_t)}{D(\boldsymbol{p}_s)}\right)^{\gamma}\left(\frac{B(\boldsymbol{p}_t)}{B(\boldsymbol{p}_s)}\right)^{-\gamma} - 1\right\}\right]^{\frac{1}{\varepsilon}} \frac{B(\boldsymbol{p}_t)}{B(\boldsymbol{p}_s)} \\
&= c(u,\boldsymbol{p}_s)\left[1 + \frac{\varepsilon w_{Ds}}{\gamma}\left\{\left(\frac{P_{Dt}}{P_{Bt}}\right)^{\gamma} - 1\right\}\right]^{\frac{1}{\varepsilon}} P_{Bt} \\
&= c(u,\boldsymbol{p}_s)\left[\left(1 - \frac{\varepsilon w_{Ds}}{\gamma}\right)P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}\cdot\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}},
\end{aligned}$$

where the second equality uses $P_{Bt} = B(\boldsymbol{p}_t)/B(\boldsymbol{p}_s)$, $P_{Dt} = D(\boldsymbol{p}_t)/D(\boldsymbol{p}_s)$, and the expenditure share in Equation (5). By the Konüs definition, Equation (1), the cost-of-living index is then

$$P_t = \left[\left(1 - \frac{\varepsilon w_{Ds}}{\gamma}\right)P_{Bt}^{\gamma} + \frac{\varepsilon w_{Ds}}{\gamma}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}\cdot\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}} = \widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}.$$

A representative level of expenditures $\bar{e}\kappa$ exists over any group of consumers, so by Muellbauer (1976, Theorem 6), group-level behavior is characterized by the same indirect utility function and expenditure function as individual-level behavior. Aggregate-level cost-of-living indices are therefore derived identically to above, with the only difference that group-level expenditure shares $\overline{w}_{Ds}$ and representative levels of expenditure $\bar{e}\kappa$ are used instead of individual-level ones. □

A.2 Proof of Lemma 1

Proof. By Proposition 1, the rate of change of the PIGL cost-of-living index is

$$\frac{P_t}{P_{t-1}} = \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}\right)^{\frac{\gamma}{\varepsilon}}\left(\frac{P_{Bt}}{P_{Bt-1}}\right)^{1-\frac{\gamma}{\varepsilon}}. \tag{A.1}$$

We want to decompose the change $\widetilde{P}_t/\widetilde{P}_{t-1}$ into price changes of the B and D baskets. To that end, recall from Equation (13) that the Hicksian expenditure share on D associated with some observed base-period expenditure share on D is

$$w_{Dt}^{h} = w_{Ds}\left(\frac{P_{Dt}}{\widetilde{P}_t}\right)^{\gamma}. \tag{A.2}$$

By the definition of $\widetilde{P}_t$, (A.2) also implies that

$$\frac{\gamma}{\varepsilon} - w_{Dt}^{h} = \left(\frac{\gamma}{\varepsilon} - w_{Ds}\right)\left(\frac{P_{Bt}}{\widetilde{P}_t}\right)^{\gamma}. \tag{A.3}$$

Now consider the following identity:

$$\left(w_{Dt}^{h} - w_{Dt-1}^{h}\right) + \left[\left(\frac{\gamma}{\varepsilon} - w_{Dt}^{h}\right) - \left(\frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}\right)\right] = 0.$$

Applying the logarithmic mean, this can be written as

$$L\left(w_{Dt}^{h}, w_{Dt-1}^{h}\right)\ln\left(\frac{w_{Dt}^{h}}{w_{Dt-1}^{h}}\right) + L\left(\frac{\gamma}{\varepsilon} - w_{Dt}^{h}, \frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}\right)\ln\left(\frac{\frac{\gamma}{\varepsilon} - w_{Dt}^{h}}{\frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}}\right) = 0. \tag{A.4}$$

Consumer optimization, as captured by Equations (A.2) and (A.3), implies that

$$\frac{w_{Dt}^{h}}{w_{Dt-1}^{h}} = \left(\frac{P_{Dt}/P_{Dt-1}}{\widetilde{P}_t/\widetilde{P}_{t-1}}\right)^{\gamma} \qquad \text{and} \qquad \frac{\frac{\gamma}{\varepsilon} - w_{Dt}^{h}}{\frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}} = \left(\frac{P_{Bt}/P_{Bt-1}}{\widetilde{P}_t/\widetilde{P}_{t-1}}\right)^{\gamma},$$

and substituting these into (A.4) yields

$$L\left(w_{Dt}^{h}, w_{Dt-1}^{h}\right)\ln\left(\frac{P_{Dt}/P_{Dt-1}}{\widetilde{P}_t/\widetilde{P}_{t-1}}\right)^{\gamma} + L\left(\frac{\gamma}{\varepsilon} - w_{Dt}^{h}, \frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}\right)\ln\left(\frac{P_{Bt}/P_{Bt-1}}{\widetilde{P}_t/\widetilde{P}_{t-1}}\right)^{\gamma} = 0.$$

We can now solve for $\widetilde{P}_t/\widetilde{P}_{t-1}$ to obtain a Sato-Vartia index over P_{Bt}/P_{Bt-1} and P_{Dt}/P_{Dt-1}:

$$\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} = \left(\frac{P_{Dt}}{P_{Dt-1}}\right)^{\phi_t}\left(\frac{P_{Bt}}{P_{Bt-1}}\right)^{1-\phi_t}, \tag{A.5a}$$

where

$$\phi_t = \frac{L\left(w_{Dt}^{h}, w_{Dt-1}^{h}\right)}{L\left(w_{Dt}^{h}, w_{Dt-1}^{h}\right) + L\left(\frac{\gamma}{\varepsilon} - w_{Dt}^{h}, \frac{\gamma}{\varepsilon} - w_{Dt-1}^{h}\right)}. \tag{A.5b}$$

Plugging Equation (A.5) into Equation (A.1) completes the proof. □

A.3 Proof of Lemma 2

Proof. Define

$$\mathcal{P}_t^L(r) \;=\; \left[\sum_{j\in J} w_{jt-1}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\frac{r}{2}}\right]^{\frac{2}{r}} \qquad \text{and} \qquad \mathcal{P}_t^P(r) \;=\; \left[\sum_{j\in J} w_{jt}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{-\frac{r}{2}}\right]^{-\frac{2}{r}},$$

such that the *quadratic-mean-of-order-r* index becomes $P_t/P_{t-1} = \sqrt{\mathcal{P}_t^L(r)\mathcal{P}_t^P(r)}$. These definitions nest the Laspeyres and Paasche indices as the special case when $r = 2$, thus motivating the L and P notation. Using the definition of $\mathcal{P}_t^L(r)$ together with the logarithmic mean, it holds that

$$\begin{aligned}
0 \;&=\; \sum_{j\in J} w_{jt-1}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\frac{r}{2}} - \left(\mathcal{P}_t^L(r)\right)^{\frac{r}{2}} \\
&=\; \sum_{j\in J} w_{jt-1}\left[\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\frac{r}{2}} - \left(\mathcal{P}_t^L(r)\right)^{\frac{r}{2}}\right] \\
&=\; \sum_{j\in J} w_{jt-1}\; L\left(\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\frac{r}{2}},\, \left(\mathcal{P}_t^L(r)\right)^{\frac{r}{2}}\right)\; \ln\left(\frac{p_{jt}/p_{jt-1}}{\mathcal{P}_t^L(r)}\right)^{\frac{r}{2}} \\
&=\; \frac{r}{2}\; \sum_{j\in J} \psi_{jt}^L\left[\ln\left(\frac{p_{jt}}{p_{jt-1}}\right) - \ln\mathcal{P}_t^L(r)\right],
\end{aligned}$$

with ψ_{jt}^L defined as in Lemma 2. Solving for $\ln\mathcal{P}_t^L(r)$, we get

$$\ln\mathcal{P}_t^L(r) \;=\; \sum_{j\in J}\frac{\psi_{jt}^L}{\sum_{i\in J}\psi_{it}^L}\ln\left(\frac{p_{jt}}{p_{jt-1}}\right).$$

Identical steps for $\mathcal{P}_t^P(r)$ yields

$$\ln\mathcal{P}_t^P(r) \;=\; \sum_{j\in J}\frac{\psi_{jt}^P}{\sum_{i\in J}\psi_{it}^P}\ln\left(\frac{p_{jt}}{p_{jt-1}}\right),$$

with ψ_{jt}^P defined as in Lemma 2. Substituting these into the overall index P_t/P_{t-1} yields

$$\frac{P_t}{P_{t-1}} \;=\; \prod_{j\in J}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\delta_{jt}}, \qquad\qquad \delta_{jt} \;=\; \frac{1}{2}\left[\frac{\psi_{jt}^L}{\sum_i\psi_{it}^L} + \frac{\psi_{jt}^P}{\sum_i\psi_{it}^P}\right],$$

and we are done. □

A.4 Proof of Proposition 4

Proof. If $\varepsilon \to 0$ and $\gamma \to 0$, the indirect utility function in Equation (4) becomes Cobb-Douglas: $V(e, \boldsymbol{p}) = \ln e - \ln \left[D(\boldsymbol{p})^{\nu} B(\boldsymbol{p})^{1-\nu} \right]$. The cost-of-living index between periods t and $t-1$ is then $P_t/P_{t-1} = \left(P_{Dt}/P_{Dt-1}\right)^{\nu} \left(P_{Bt}/P_{Bt-1}\right)^{1-\nu}$ by Lemma 1 and Equation (5), where the weights are time-invariant expenditure shares: $\nu = w_D$ and $1-\nu = w_B$. Let the subindex for bundle $C \in \{B, D\}$ be of a Törnqvist form, such that $\ln\left(P_{Ct}/P_{Ct-1}\right) = \sum_j \delta_{jt}^{C} \ln\left(p_{jt}/p_{jt-1}\right)$, with $\delta_{jt}^{C} = \left(w_{jt}^{C} + w_{jt-1}^{C}\right)/2$ for all j in basket C. Substituting the Törnqvist subindex for C into the overall index, the weight on good j in bundle C consequently becomes $w_C\, \delta_{jt}^{C} = (w_{jt} + w_{jt-1})/2$, since $w_j = w_C\, w_j^{C}$ holds by definition under Assumption 1. These are standard Törnqvist weights, thus proving the result. $\square$

Appendix B Extensions

B.1 Allowing for Heterogeneity in Tastes

Redding and Weinstein (2020) stress the importance of accounting for heterogeneity in tastes for the cost of living and it is possible to extend the baseline framework to allow for this. Following Cravino, Levchenko and Rojas (2022), let the preferences of consumer h be characterized by an indirect utility function of the form

$$V_h(e_h, \boldsymbol{p}) \;=\; \frac{1}{\varepsilon}\left[\left(\frac{e_h}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] \;-\; \frac{\nu_h}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right], \tag{B.1}$$

where the only difference to the benchmark PIGL specification is that we allow for a time-invariant taste parameter ν_h that varies across consumers. As before, the expenditure share of the latent good with price function $D(\cdot)$ is given by Roy's identity as

$$w_{Dh} \;=\; \nu_h \left(\frac{B(\boldsymbol{p})}{e_h}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma},$$

and the corresponding aggregate expenditure share over a measure N of consumers with weights μ_h is now

$$\overline{w}_D \;=\; \bar{\nu}\left(\frac{B(\boldsymbol{p})}{\bar{e}\kappa}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}, \qquad \text{where} \qquad \kappa \;=\; \left[\int_0^N \mu_h \,\frac{\nu_h}{\bar{\nu}}\left(\frac{e_h}{\bar{e}}\right)^{-\varepsilon} dh\right]^{-\frac{1}{\varepsilon}},$$

and where $\bar{\nu} = \frac{1}{N}\int_0^N \nu_h \, dh$ denotes the average taste. A representative agent with expenditure level $\bar{e}\kappa$ therefore exists and incorporates any deviations from the mean taste level. By Muellbauer (1976, Theorem 6), aggregate-level behavior is therefore characterized by an expenditure function

$$c(u^{RA}, \boldsymbol{p}) \;=\; \left[1 \,+\, \varepsilon\left(u^{RA} \,+\, \frac{\bar{\nu}}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}),$$

for some corresponding representative utility level u^{RA}. This expenditure function is independent of individual taste parameters ν_h. We can therefore follow the same steps as for the benchmark PIGL specification to derive an identical price index. Again, this index is only a function of the base-period expenditure share for the D basket, price indices P_{Dt} and P_{Bt}, and the parameters ε and γ. Heterogeneity in the taste parameters ν_h only affect the price index indirectly to the extent that they affect expenditure shares. Whenever base-period expenditure shares are observed in the data, there is no need to know these individual tastes to compute the price index. Since this holds for any measure N of consumers, it also for singleton groups, so the price index also applies at the individual level.

B.2 Allowing for Hump-Shaped Expenditure Shares

Banks, Blundell and Lewbel (1997) stress the importance of allowing for hump-shaped expenditure shares to match the microeconomic data and it is possible to extend the baseline framework to allow for this at the individual level. Following Alder, Boppart and Müller (2022), let preferences be characterized by an indirect utility function of the form

$$V(e,\boldsymbol{p}) \;=\; \frac{1}{\varepsilon}\left[\left(\frac{e-A(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\varepsilon}-1\right]-\frac{\nu}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}-1\right], \tag{B.2}$$

where the only difference to the benchmark PIGL specification is the addition of a linearly homogeneous function $A(\boldsymbol{p})$ of prices. The expenditure shares of the three latent goods with price functions $A(\cdot)$, $B(\cdot)$ and $D(\cdot)$ are given by Roy's identity as

$$w_A \;=\; \frac{A(\boldsymbol{p})}{e}, \tag{B.3}$$

$$w_B \;=\; \left(1-\frac{A(\boldsymbol{p})}{e}\right)\left[1-\nu\left(\frac{B(\boldsymbol{p})}{e-A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\right], \tag{B.4}$$

$$w_D \;=\; \left(1-\frac{A(\boldsymbol{p})}{e}\right)\nu\left(\frac{B(\boldsymbol{p})}{e-A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}. \tag{B.5}$$

The shares w_j^A, w_j^B and w_j^D of total A, B and D expenditures allocated to an individual good j are given as before by $w_j^C = p_j C_j(\boldsymbol{p})/C(\boldsymbol{p})$, $C \in \{A, B, D\}$. Together with Equations (B.3) to (B.5), this implies an expenditure share w_j of good j in total expenditures of the form

$$\begin{aligned} w_j \;=\; p_j \Bigg\{ &\frac{A(\boldsymbol{p})}{e}\,\frac{A_j(\boldsymbol{p})}{A(\boldsymbol{p})} \\ &+\left(1-\frac{A(\boldsymbol{p})}{e}\right)\left[\frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})}+\left(\frac{D_j(\boldsymbol{p})}{D(\boldsymbol{p})}-\frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})}\right)\nu\left(\frac{B(\boldsymbol{p})}{e-A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\,\right]\Bigg\}. \end{aligned} \tag{B.6}$$

Since the first term on the right-hand side of (B.6) is decreasing in e while the second term can be either increasing or decreasing in e, this allows for expenditure shares that are nonmonotonic in expenditures. The derivation of the exact price index of (B.2) is virtually identical to the PIGL case. The corresponding expenditure function of (B.2) is

$$c(u,\boldsymbol{p}) \;=\; \left[1+\varepsilon\left(u+\frac{\nu}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}-1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}) \,+\, A(\boldsymbol{p}).$$

Suppose again that the reference utility u corresponds to the observed consumption expenditure level in a base period s, such that $u \equiv V(e_s, \boldsymbol{p}_s)$ and $c(u, \boldsymbol{p}_s) = e_s$. Substituting the period-s indirect utility function (B.2) into a period-t expenditure function, rearranging terms, and using $P_{Ct} = C(\boldsymbol{p}_t)/C(\boldsymbol{p}_s)$ together with Equations (B.3) to (B.5) yields

$$c(u,\boldsymbol{p}_t) \;=\; e_s\left\{(1-w_{As})\left[\left(1-\frac{\varepsilon}{\gamma}\frac{w_{Ds}}{1-w_{As}}\right)P_{Bt}^{\gamma}+\frac{\varepsilon}{\gamma}\frac{w_{Ds}}{1-w_{As}}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}}P_{Bt}^{1-\frac{\gamma}{\varepsilon}}+w_{As}P_{At}\right\},$$

and it follows that the price index is

$$P_t = (1 - w_{As})\widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}} + w_{As}P_{At}$$

where

$$\widetilde{P}_t = \left[\left(1 - \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}}\right) P_{Bt}^{\gamma} + \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} P_{Dt}^{\gamma} \right]^{\frac{1}{\gamma}}.$$

This index is a direct generalization of the PIGL cost-of-living index and, as before, is computable given base-period expenditure shares w_{As}, w_{Ds}, price indices P_{At}, P_{Bt}, P_{Dt} and parameter values for ε and γ. Proposition 3 still applies: quasi-separability between goods that are always necessities, goods that are luxuries for low expenditures and necessities for high expenditures, and goods that are always luxuries, together with appropriate choices for P_{At}, P_{Bt} and P_{Dt}, reduces estimation to the two parameters ε and γ which are readily obtained from Equations (B.3) and (B.5).

Unlike the baseline framework, however, these preferences do not easily aggregate. Aggregate expenditure shares over a measure N of consumers with weights μ_h are now

$$\begin{aligned}
\overline{w}_A &= \frac{A(\boldsymbol{p})}{\bar{e}\kappa_A}, \\
\overline{w}_B &= \left(1 - \frac{A(\boldsymbol{p})}{\bar{e}\kappa_A}\right) \left[1 - \nu \left(\frac{B(\boldsymbol{p})}{(\bar{e}\kappa_A - A(\boldsymbol{p}))\,\kappa_D}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\right], \\
\overline{w}_D &= \left(1 - \frac{A(\boldsymbol{p})}{\bar{e}\kappa_A}\right) \nu \left(\frac{B(\boldsymbol{p})}{(\bar{e}\kappa_A - A(\boldsymbol{p}))\,\kappa_D}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma},
\end{aligned}$$

where

$$\begin{aligned}
\kappa_A &= \left[\int_0^N \mu_h \left(\frac{e_h}{\bar{e}}\right)^{-1} dh \right]^{-1}, \\
\kappa_D &= \left[\int_0^N \mu_h \frac{\bar{e}\kappa_A}{e_h} \left(\frac{e_h - A(\boldsymbol{p})}{\bar{e}\kappa_A - A(\boldsymbol{p})}\right)^{1-\varepsilon} dh \right]^{-\frac{1}{\varepsilon}}.
\end{aligned}$$

Unlike the PIGL case, there is no representative level of expenditure in Muellbauer's (1975, 1976) sense: the expenditure level that induces the average expenditure share for A is $\bar{e}\kappa_A$ while the expenditure level e^{RA} that induces the average shares for B and D is implicitly determined by $\left(e^{RA} - A(\boldsymbol{p})\right)^{1-\varepsilon} / \, e^{RA} = \left(\bar{e}\kappa_A - A(\boldsymbol{p})\right)^{1-\varepsilon} \kappa_D^{-\varepsilon} / \, \bar{e}\kappa_A$, and these are generally not the same. Therefore, it is not possible to bake in both parameters κ_A and κ_D into some representative level of expenditure and proceed as for an individual consumer. A representative *agent* with expenditure level $\bar{e}\kappa_A$ and taste parameter κ_D exists, however. The expenditure function of this representative agent is now

$$c(u^{RA}, \boldsymbol{p}) = \left[1 + \varepsilon\left(u^{RA} + \frac{\nu\kappa_D^{-\varepsilon}}{\gamma}\left\{ \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}) + A(\boldsymbol{p}),$$

for some corresponding representative utility level u^{RA}. Similar steps as before gives an aggregate price index of the same form as above, $P_t = (1 - \overline{w}_{As})\widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}} + \overline{w}_{As} P_{At}$, but with $\widetilde{P}_t$ now given by

$$\widetilde{P}_t = \left[\left(1 - \frac{\varepsilon}{\gamma} \left(\frac{\kappa_{Dt}}{\kappa_{Ds}} \right)^{-\varepsilon} \frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}} \right) P_{Bt}^{\gamma} + \frac{\varepsilon}{\gamma} \left(\frac{\kappa_{Dt}}{\kappa_{Ds}} \right)^{-\varepsilon} \frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}} P_{Dt}^{\gamma} \right]^{\frac{1}{\gamma}}.$$

Thus, to compute aggregate price indices, we either need to know the inequality measures κ_A and κ_D for all time periods considered, or we need to impose the (likely strong) assumption that these measures remain constant over time for *all* groups considered.

Appendix C Additional Tables and Figures

Table C.1. CEX-CPI crosswalk.

	CEX category	**CPI name**	**CPI code**
1	Food at home	Food at home	SAF11
2	Food away from home	Food away from home	SEFV
3	Alcoholic beverages	Alcoholic beverages	SAF116
4	Rented dwellings	Rent of primary residence	SEHA
5	Owned dwellings[a]	Owners' equivalent rent of primary residence	SEHC
6	Other lodging	Lodging while out of town[b]	MUUR0000SE2102
		Lodging away from home[b]	SEHB
7	Utilities	Household energy	SAH21
8	Water	Water and sewerage maintenance	SEHG01
9	Phone	Communication	SAE2
10	Household O&F[c]	Household furnishings and operations	SAH3
11	Apparel	Apparel	SAA
12	Gasoline	Motor fuel	SETB
13	Other vehicle expenses	Motor vehicle maintenance and repair	SETD
		Motor vehicle insurance	SETE
		Motor vehicle fees	SETF
14	Public transportation	Public transportation	SETG
15	Health	Medical care	SAM
16	Entertainment	Recreation	SAR
17	Personal care	Personal care	SAG1
18	Reading	Recreational reading materials	SERG
19	Education	Education and communcation	SAE
20	Tobacco	Tobacco and smoking products	SEGA
21	Other expenses	Miscellaneous personal services	SEGD

Notes. The CEX categories follow the hierarchical groupings defined by the BLS. CPIs are non-seasonally adjusted nationwide data for urban consumers.
[a] Rental equivalence value of owned dwellings as reported by the households.
[b] "Lodging away from home" from 1995–1997 and "lodging while out of town" afterwards.
[c] Operations and furnishing, includes "household operations", "housekeeping supplies" and "household furnishings and equipment".

Table C.2. Estimated Engel curve slopes and classification of expenditure categories.

Category and classification	(1)	(2)	(3)	(4)	(5)	(6)
Luxuries						
Owned dwellings	1.757	1.629	1.631	1.707	0.0001800	9.461
	(0.019)	(0.020)	(0.020)	(0.022)	(0.0000072)	(0.131)
Household operations and furnishings	0.690	0.725	0.724	0.740	0.0001470	4.762
	(0.010)	(0.010)	(0.010)	(0.011)	(0.0000041)	(0.084)
Entertainment	0.386	0.405	0.404	0.417	0.0000604	2.427
	(0.006)	(0.006)	(0.006)	(0.007)	(0.0000024)	(0.048)
Other lodging	0.303	0.294	0.294	0.300	0.0000532	1.838
	(0.005)	(0.004)	(0.004)	(0.005)	(0.0000017)	(0.033)
Food away from home	0.299	0.323	0.324	0.348	0.0000415	1.957
	(0.005)	(0.005)	(0.005)	(0.006)	(0.0000016)	(0.035)
Education	0.280	0.281	0.281	0.286	0.0000560	1.825
	(0.006)	(0.006)	(0.006)	(0.007)	(0.0000023)	(0.048)
Health	0.215	0.163	0.166	0.230	0.0000176	1.265
	(0.007)	(0.007)	(0.007)	(0.008)	(0.0000015)	(0.044)
Public transport	0.137	0.139	0.139	0.120	0.0000197	0.692
	(0.003)	(0.003)	(0.003)	(0.003)	(0.0000008)	(0.019)
Other vehicle expenditures	0.124	0.135	0.132	0.153	0.0000056	0.863
	(0.006)	(0.006)	(0.006)	(0.007)	(0.0000011)	(0.039)
Other expenses	0.105	0.102	0.104	0.111	0.0000183	0.682
	(0.002)	(0.002)	(0.002)	(0.002)	(0.0000006)	(0.014)
Apparel	0.098	0.122	0.118	0.116	0.0000205	0.689
	(0.004)	(0.004)	(0.004)	(0.004)	(0.0000015)	(0.028)
Alcoholic beverages	0.044	0.047	0.047	0.046	0.0000061	0.255
	(0.001)	(0.001)	(0.001)	(0.001)	(0.0000003)	(0.008)
Personal care	0.031	0.032	0.031	0.031	0.0000038	0.170
	(0.001)	(0.001)	(0.001)	(0.001)	(0.0000002)	(0.006)
Reading	0.028	0.027	0.027	0.026	0.0000027	0.139
	(0.001)	(0.001)	(0.001)	(0.001)	(0.0000001)	(0.003)
Necessities						
Water	−0.045	−0.051	−0.050	−0.037	−0.0000064	−0.220
	(0.002)	(0.002)	(0.002)	(0.002)	(0.0000002)	(0.010)
Tobacco	−0.158	−0.166	−0.167	−0.146	−0.0000194	−0.843
	(0.003)	(0.003)	(0.003)	(0.003)	(0.0000007)	(0.023)
Phone	−0.186	−0.186	−0.186	−0.182	−0.0000279	−1.065
	(0.003)	(0.003)	(0.003)	(0.003)	(0.0000007)	(0.017)
Category dummies	✓	✓	✓	✓	✓	✓
5-year age-bin fixed effects		✓	✓	✓	✓	✓
Year fixed effects			✓	✓	✓	✓
State fixed effects				✓	✓	✓
Adjusted R^2	0.579	0.591	0.597	0.605	0.584	0.606
Observations	1,562,211	1,542,765	1,542,765	1,351,413	1,351,413	1,351,413

Continued on the next page

Table C.2. Estimated Engel curve slopes and classification of expenditure categories. (Cont.)

Category and classification	(1)	(2)	(3)	(4)	(5)	(6)
Gasoline	−0.219	−0.216	−0.214	−0.168	−0.0000335	−0.958
	(0.004)	(0.004)	(0.004)	(0.004)	(0.0000009)	(0.027)
Utilities	−0.362	−0.380	−0.381	−0.329	−0.0000445	−1.904
	(0.004)	(0.004)	(0.004)	(0.004)	(0.0000012)	(0.025)
Rented dwellings	−1.496	−1.390	−1.387	−1.756	−0.0002220	−10.070
	(0.018)	(0.018)	(0.018)	(0.020)	(0.0000063)	(0.115)
Food at home	−2.031	−2.035	−2.037	−2.013	−0.0002780	−11.970
	(0.012)	(0.012)	(0.012)	(0.014)	(0.0000066)	(0.082)
Category dummies	✓	✓	✓	✓	✓	✓
5-year age-bin fixed effects		✓	✓	✓	✓	✓
Year fixed effects			✓	✓	✓	✓
State fixed effects				✓	✓	✓
Adjusted R^2	0.579	0.591	0.597	0.605	0.584	0.606
Observations	1,562,211	1,542,765	1,542,765	1,351,413	1,351,413	1,351,413

Notes. Standard errors clustered at the household level in parentheses. Columns (1) to (4) show the coefficient estimates from a weighted least square regression of expenditure shares (in percent) on the expenditure decile interacted with expenditure category dummies using the CEX household sampling weights. All fixed effects are by expenditure category. Columns (5) and (6) show coefficient estimates for the same regression but using the expenditure level and log expenditure level instead of the expenditure decile.

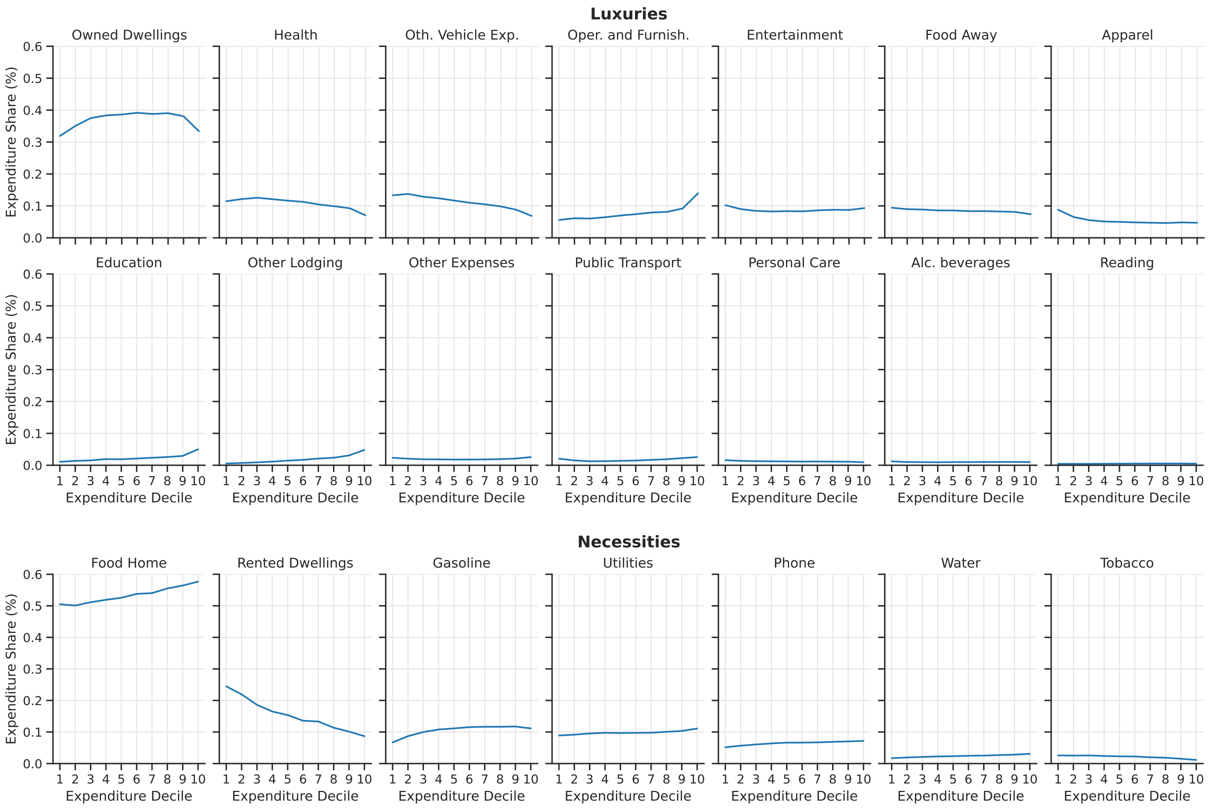

FIGURE C.1. Expenditure shares within luxuries and necessities by expenditure decile.

Notes. The figure shows the expenditure share of each expenditure category within the group of luxuries and necessities averaged over all years. If quasi-separability held perfectly, the expenditure share would be constant for all expenditure deciles. Thus, any slope different from zero highlights the residual nonhomotheticity within luxuries and necessities.

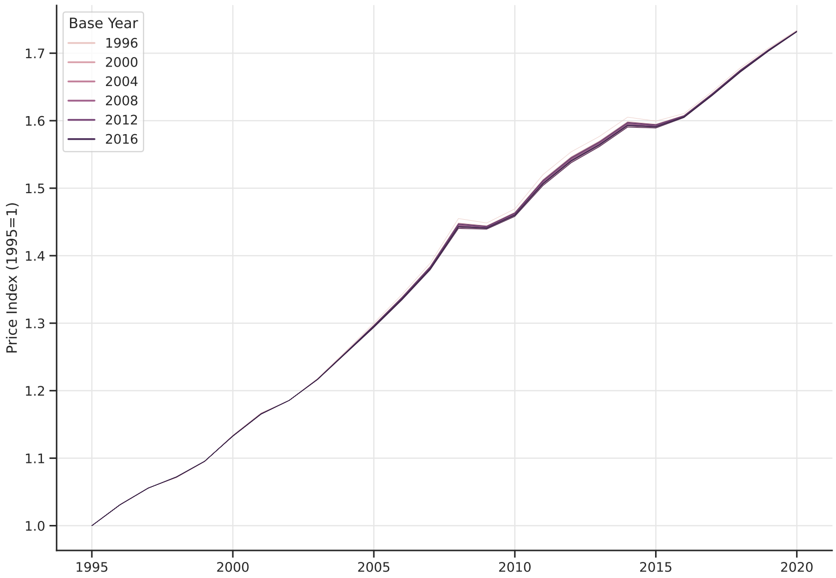

FIGURE C.2. PIGL representative agent Generalized Sato-Vartia price index for different base years.

Notes. The price index is calculated under quasi-separability. Each line represents the representative agent price index for a different base year, but normalized to one in 1995.

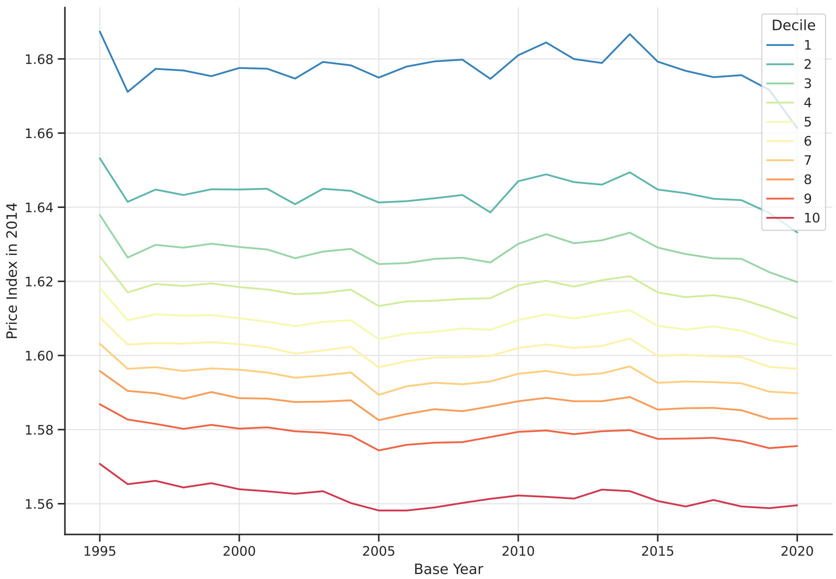

FIGURE C.3. Generalized Sato-Vartia price index in 2014 by expenditure decile for different base years.

Notes. The price index is calculated under quasi-separability. The horizontal axis describes the base year of the price index and the vertical axis the respective value of the price index in 2014. Price indices are all normalized to one in 1995. The price index for each expenditure decile is calculated as the price index of the PIGL representative agent over households within each respective decile.

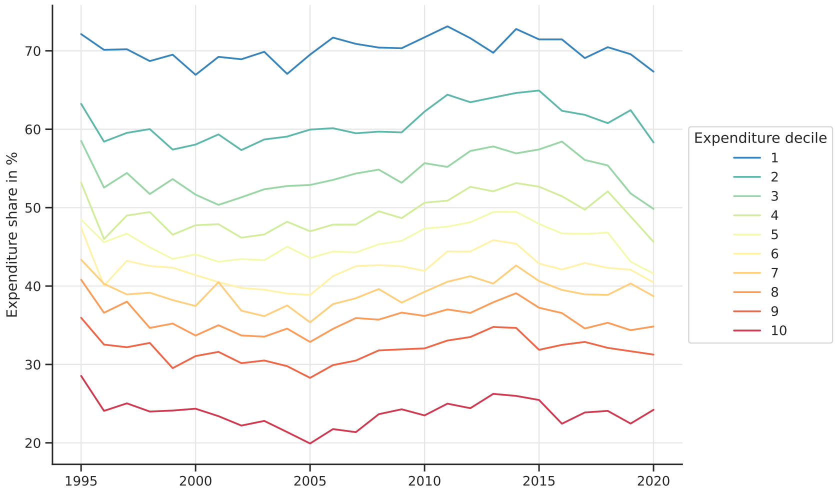

Figure C.4. Expenditure share on necessities, by expenditure decile.

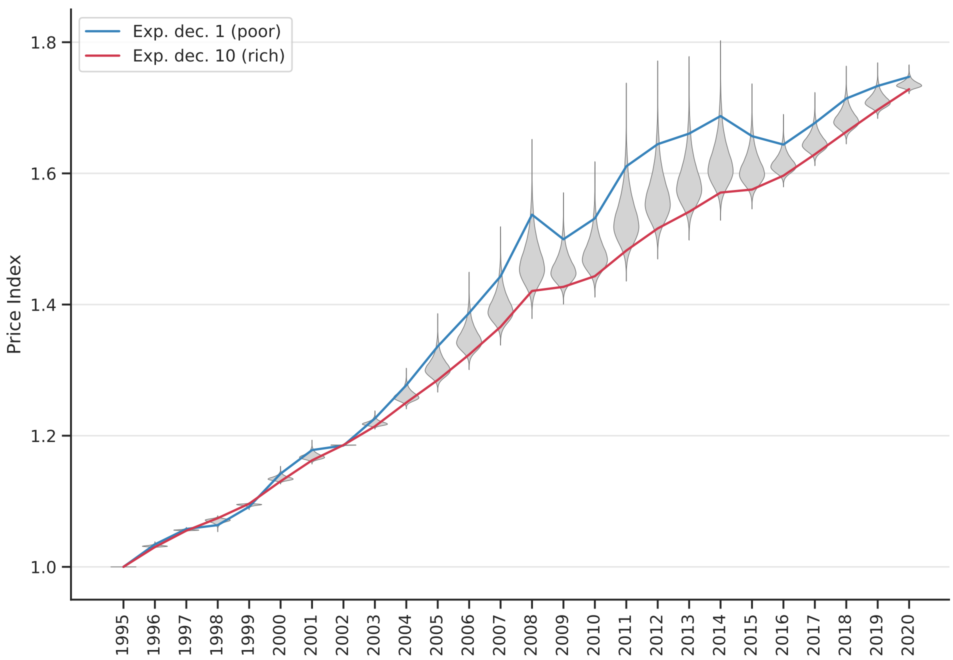

Figure C.5. Distribution of the Generalized Sato-Vartia price index under quasi-separability.

Notes. The gray shaded area shows the kernel density estimate of the distribution of Generalized Sato-Vartia price indices. The index is calculated for each household in the sample of 1995. The blue line shows the PIGL RA price index for the poorest 10 percent. The red line shows the PIGL RA price index for the richest 10 percent.

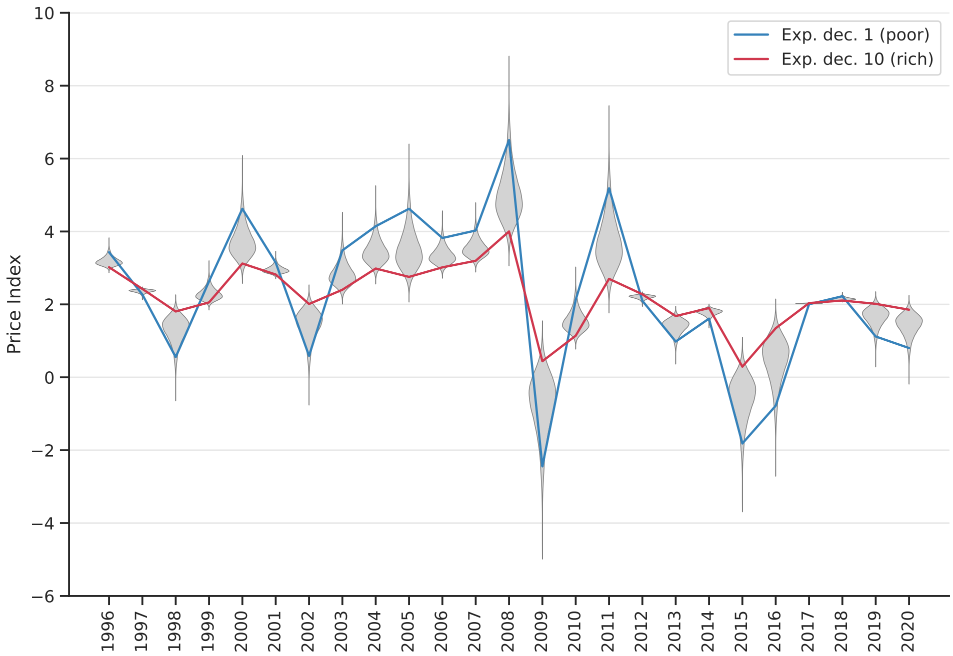

FIGURE C.6. Distribution of Generalized Sato-Vartia inflation under quasi-separability.

Notes. The gray shaded area shows the kernel density estimate of the distribution of inflation rates. The inflation is calculated for each household in the sample of 1995. The blue line shows the PIGL RA inflation for the poorest 10 percent. The red line shows the PIGL RA inflation for the richest 10 percent.

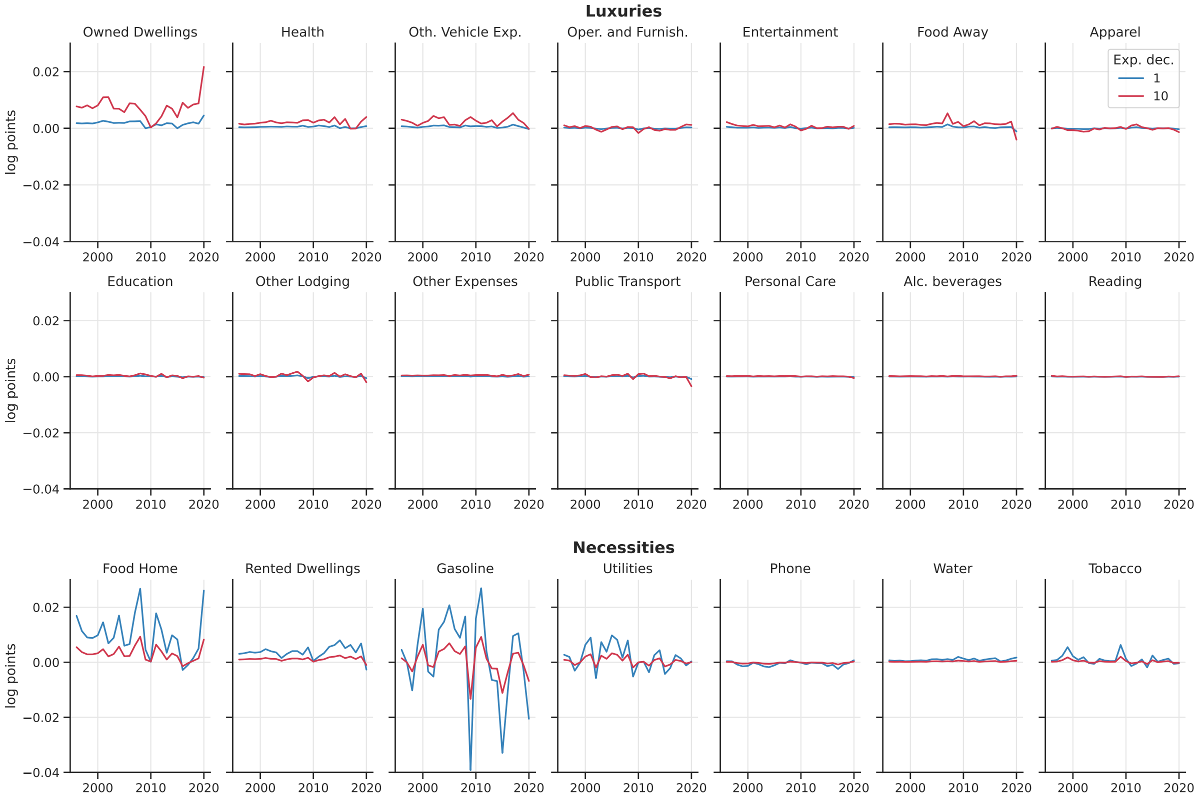

FIGURE C.7. Inflation decomposition by expenditure categories.

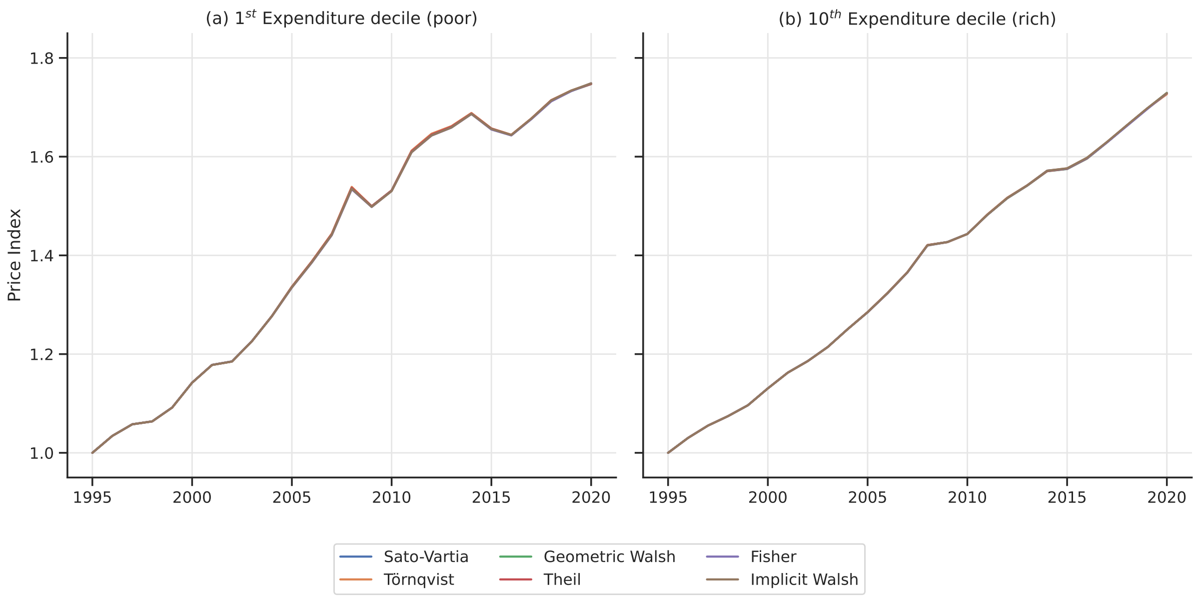

FIGURE C.8. Comparison of different generalized superlative price indices.

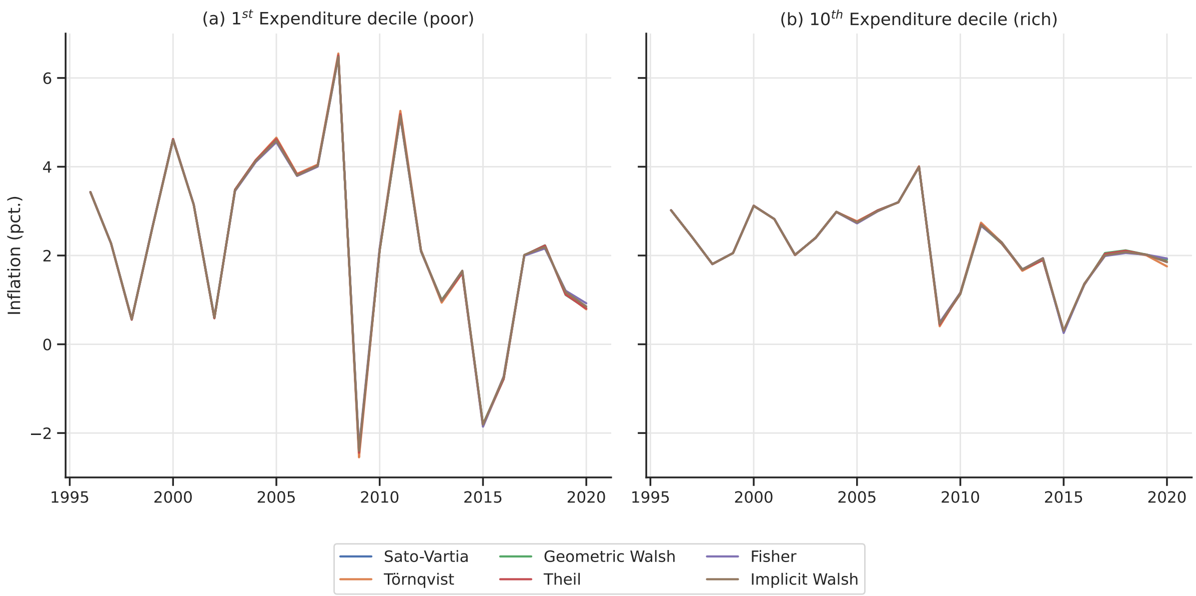

FIGURE C.9. Comparison of inflation across different generalized superlative indices.

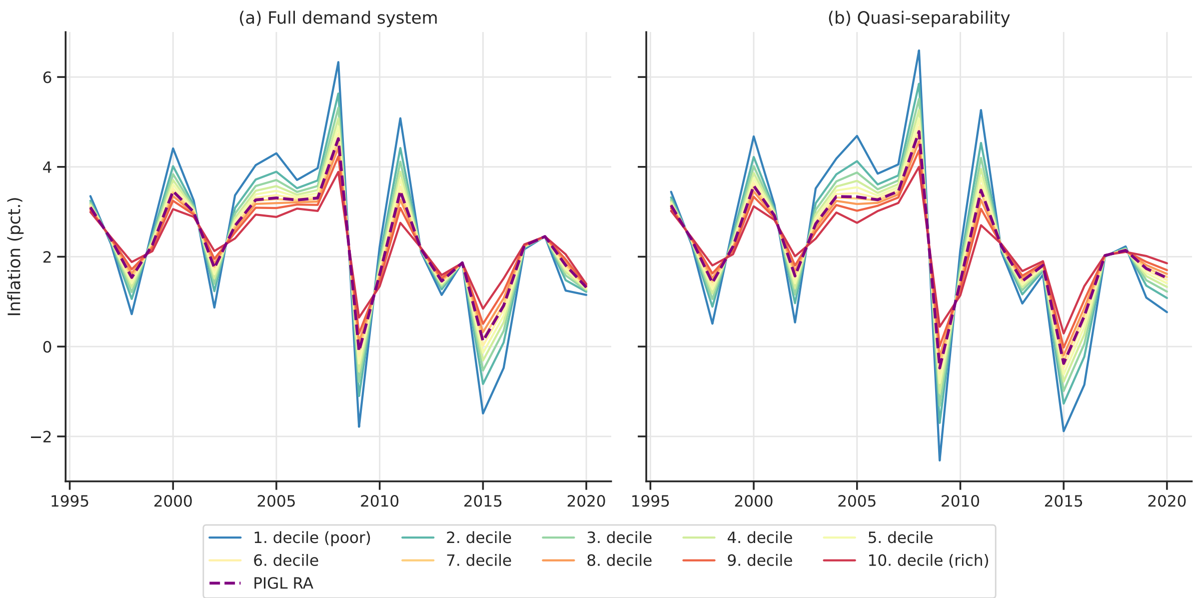

FIGURE C.10. Comparison of Generalized Sato-Vartia inflation between the full demand system estimation and under quasi-separability.

Appendix References

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