A Nonhomothetic Price Index and Inflation Heterogeneity*

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Abstract

We derive a microfounded, nonhomothetic generalization of all known superlative price indices, including the Fisher, the Törnqvist, and the Sato-Vartia indices. The index varies continuously along the consumption distribution, aggregates consistently across heterogeneous households and largely avoids the need for estimation. In an empirical application to the United States using CEX-CPI data for the period 1995–2020, we find: (i) poor and rich households experience on average the same inflation rate; but (ii) inflation for the poorest decile is more than 2.5 times as volatile as that of the richest decile; and (iii) this higher volatility primarily stems from a larger exposure to price changes in food, gasoline and utilities. Our findings contrast with papers that construct standard price indices for different consumer groups. We show that the inflation inequality uncovered in these analyses may be a spurious result of failing to purge the underlying price indices from a bias owing to income effects on consumer behavior.

Keywords: cost of living, inequality, nonhomotheticity, superlative price index.

JEL codes: C43, D11, D12, E31, I30.

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1 Introduction

Do changes in the cost of living vary with income? Conventional price indices that are used to measure inflation cannot answer this question because they rely fully on the assumption of homothetic preferences. That is, consumers are assumed to make identical allocations, regardless of income level. Yet, one of the oldest empirical economic facts, dating back to at least Engel (1857), is that consumption patterns differ systematically between rich and poor consumers. In other words, preferences are ***not*** homothetic. Differences in consumption bundles raises the possibility for inflation inequality, with potential implications for any area in which price changes matter, not least for the measurement of real incomes and monetary policy. Several recent papers investigate this issue by either computing homothetic price indices for separate income groups or using nonparametric algorithms. The question of how to consistently incorporate nonhomothetic preferences into conventional and easy-to-use price index formulas, however, remains unsolved.

The goal of this paper is to tackle this problem head-on. In doing so, we make three main contributions. First, we derive a cost-of-living index that is consistent with nonhomothetic consumer demand theory. In its most general form, this index nests all known superlative price indices as special cases, including the Fisher (1922), the Törnqvist (1936), and the Sato (1976) and Vartia (1976) indices. Second, we outline a feasible strategy to compute these price indices without being at the mercy of estimating entire consumer demand systems. Instead, under a relatively mild assumption, estimation reduces to two parameters which are identified from a single equation. Third, we implement this approach using US consumption and CPI data to investigate inflation heterogeneity over the last quarter century.

Our framework allows for a characterization of the cost of living for the full consumption distribution as well as at the aggregate level. We achieve this by deriving cost-of-living indices from a specification of Muellbauer's (1975, 1976) "price independent generalized linearity" (PIGL) preferences that has recently gained popularity in the structural transformation literature.[1] These preferences are nonhomothetic but maintain tractable aggregation properties that allow us to account for consumer heterogeneity. Like many conventional price indices, we show that the PIGL preferences induce cost-of-living indices that are weighted geometric means of individual price changes. Unlike their homothetic counterparts, however, the weights on these price changes vary systematically and monotonically across the consumption distribution. Specifically, richer households allocate higher weights to price changes of luxury goods. Changes in the cost of living are consequently allowed to differ with the consumption expenditure level.

Due to the nonhomothetic nature of the underlying PIGL preferences, the cost-of-living index in its most general form is not directly computable without estimating a complete consumer demand system. We overcome this hurdle by imposing a key assumption: that preferences are quasi separable into necessities and luxuries. That is, we assume that prices of individual goods are separable into a necessity bundle and a luxury bundle in the expenditure function. Nonhomotheticity runs across these two bundles whereas behavior within each bundle is homothetic, so the price indices for each individual bundle can be obtained with standard price index formulas. Under the quasi separability assumption, the cost-of-living index then reduces

[1] See for instance Boppart (2014), Alder, Boppart and Müller (2022), and Cravino, Levchenko and Rojas (2022).

to observed price changes and expenditure shares and only two unknown parameters, which are readily estimated by linear as well as nonlinear regression methods. As in for instance Wachter and Yogo (2010) and Orchard (2022), the classification of individual goods as "necessity" or "luxury" is straightforwardly done by investigating Engel curves. The cost-of-living index still nests homothetic price indices as special cases, so the assumption of quasi-separable preferences is not a hard restriction when using homotheticity as our point of departure. In our empirical application, we also show that it is well justified in the data.

We illustrate this method with an empirical analysis of US inflation heterogeneity over the years 1995 to 2020. In this exercise, we consider twenty-one consumption good categories from the Consumer Expenditure Survey that we match with corresponding CPI sub-indices. We obtain two key results. First, households in the first consumption decile ("the poor") experienced similar inflation on average as households in the tenth consumption decile ("the rich") between 1995 and 2020, although substantial differences arise in the period around the Great Recession. In particular, poor households faced a 0.37 percentage point higher average annual inflation rate between 2004 and 2015. Second, while the average inflation *rate* is similar between the poor and the rich, inflation *volatility* is more than 2.5 times higher for the poor, as measured by the standard deviation of the annual inflation rate over time within each consumption decile.

To understand the driving factors behind these results, we decompose the changes in the cost of living by product categories. This generates a third empirical insight: the higher inflation volatility for the poor primarily stems from this group having a larger exposure to price changes in food, gasoline, and utilities. A behavioral decomposition also reveals that the overall development of the cost-of-living indices is almost entirely driven by mechanical price changes on the base-period consumption baskets. Substitution behavior, as relative prices change, plays only a minor role. Yet, both differential base-period reference baskets and differential substitution are significant drivers in explaining the cost-of-living index *differences* between groups, as poor households substitute away from expensive goods to a larger degree than the rich.

Our empirical application adds to a strand of literature on the measurement of inflation inequality which dates back at least to the 1950s (see for instance Muellbauer, 1974, and references therein), with recent advances surveyed by Jaravel (2021). The bulk of this literature approximates nonhomotheticity by computing standard price indices separately for different income or consumer groups.[2] The price indices obtained with this method are nevertheless homothetic within each group and therefore subject to similar criticism as above. More importantly, by relying on changes in *observed* consumer behavior over time, these indices also capture income effects related to changes in the standard of living within each group. These income effects generate biases relative to the true cost-of-living indices, which should only reflect the changes in consumption expenditures needed to maintain fixed standards of living; see Oulton (2008) for further discussion. The theoretically consistent approach that we propose avoids this bias.

In the final parts of the paper, we evaluate this bias by applying a similar approach as in previous work and compute homothetic price indices for each consumption decile using the same data as in our main analysis. Compared to our baseline results, we find that this group-specific approach

[2] Recent papers employing this approach include Hobijn and Lagakos (2005), McGranahan and Paulson (2005), Broda and Romalis (2009), Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Argente and Lee (2021), Klick and Stockburger (2021), Lauper and Mangiante (2021), and Orchard (2022).

increases inequality in average inflation rates but decreases the differences in inflation volatility. Specifically, the difference in the average annual inflation rate between the bottom and top consumption deciles increases by 0.3 percentage points compared to our baseline findings. The magnitude of this bias is similar to previous measurements of inflation inequality in the United States for comparable time periods.

Besides the empirical contribution, this paper also falls within an old literature on the economic approach to price index theory following Konüs (1939), Samuelson and Swamy (1974), Diewert (1976, 1978), Feenstra (1994), Redding and Weinstein (2020), and many others, whereby cost-of-living indices are derived from consumer theory via the expenditure function.[3] Central to this line of research is Diewert's notion of a *superlative* price index, which includes indices that are exact for some homothetic expenditure functions and can approximate other homothetic indices to the second order. The Fisher, the Törnqvist and the Sato-Vartia indices are all known to satisfy this property (see Diewert, 1976, for the former two and Barnett and Choi, 2008, for the latter). Our paper provides a nonhomothetic generalization of these and all other currently known indices within this class.

Nonhomothetic preferences generally receive limited attention in the construction of implementable price index formulas. Efforts to measure changes in the cost of living and consumer welfare under nonhomothetic behavior instead focus on nonparametric methods (Atkin *et al.*, 2020; Baqaee, Burstein and Koike-Mori, 2022; Jaravel and Lashkari, 2022), numerical algorithms (Vartia, 1983; Dumagan and Mount, 1997; Oulton, 2008, 2012), or estimations of fully specified consumer demand systems (Banks, Blundell and Lewbel, 1997; Almås and Kjelsrud, 2017). Two exceptions are Feenstra and Reinsdorf (2000) and Redding and Weinstein (2020). Feenstra and Reinsdorf obtain a Divisia index approximation for Deaton and Muellbauer's (1980a) Almost Ideal Demand System.[4] Divisia indices, however, are known to differ from true cost-of-living indices when preferences are nonhomothetic. Redding and Weinstein derive a theoretical price index for the nonhomothetic CES specification considered by for example Matsuyama (2019) and Comin, Lashkari and Mestieri (2021). In contrast to our index, the nonhomothetic CES specification does not consistently aggregate across heterogeneous consumers and (to the best of our knowledge) provides no easy implementation empirically without being forced to estimate all parameters in the utility function.

The paper proceeds as follows. Section 2 covers the theoretical framework, derives the nonhomothetic price index and shows how all superlative price indices can be generalized to a nonhomothetic setting. Section 3 outlines the strategy of our empirical implementation and discusses the assumption we make to render the demand system estimation feasible. Section 4 explains the data we employ, classifies goods into necessities and luxuries and reports estimates for the two preference parameters. Section 5 reports the main empirical results, while Section 6 compares the price index with previous standard methods to evaluate inflation inequality. Section 7 compares the main results to traditional demand system estimation and Section 8 concludes.

[3] Diewert (1993) surveys the early stages of this literature, which is far too large for us to do justice to here.

[4] The Divisia index decomposes a change in consumption expenditure e into price and quantity indices with equivalent forms: $e_t/e_{t-1} = P_t Q_t$, where $\ln P_t \equiv \int_{t-1}^{t} \sum_j w_{j\tau} \frac{d \ln p_{j\tau}}{d\tau} d\tau$ and $\ln Q_t \equiv \int_{t-1}^{t} \sum_j w_{j\tau} \frac{d \ln q_{j\tau}}{d\tau} d\tau$ are line integrals over price and quantity changes using expenditure shares $w_{j\tau}$ as weights.

2 A Nonhomothetic Cost-of-Living Index

The framework we consider is one where consumers maximize utility over a set of goods J with a corresponding price vector $\boldsymbol{p}$ and where we wish to investigate the change in the cost of living between a period t and some base period s. In what follows, we drop time subscripts whenever possible to simplify notation, as long as this causes no confusion. The minimum consumption expenditure e required to obtain some utility level u when faced by the price vector $\boldsymbol{p}$ is given by the expenditure function $e = c(u, \boldsymbol{p})$. Following Konüs (1939), we define a cost-of-living index in period t relative to base period s to be the ratio of minimum expenditures required to maintain a constant utility level:

$$P(u, \boldsymbol{p}_t, \boldsymbol{p}_s) \equiv \frac{c(u, \boldsymbol{p}_t)}{c(u, \boldsymbol{p}_s)}. \tag{1}$$

Hereinafter we typically leave the arguments of the cost-of-living index implicit and simply write $P_t = c(u, \boldsymbol{p}_t)/c(u, \boldsymbol{p}_s)$.

2.1 The Homothetic Case

In general, the Konüs cost-of-living index (1) depends on the reference standard of living u as well as the prices in the two periods. Samuelson and Swamy (1974) show that independence of u occurs if and only if we consider the special case of homothetic preferences. Suppose for instance that consumer preferences are characterized by an indirect utility function of the standard homothetic CRRA form,

$$V(e, \boldsymbol{p}) = \frac{1}{\varepsilon}\left[\left(\frac{e}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right], \tag{2}$$

where $B(\boldsymbol{p})$ is a linearly homogenous function of prices and ε is the coefficient of relative risk aversion. Inverting the utility function to obtain the expenditure function $c(u, \boldsymbol{p}) = (1 + \varepsilon u)^{1/\varepsilon} B(\boldsymbol{p})$ and using Equation (1), we get

$$P_t = \frac{B(\boldsymbol{p}_t)}{B(\boldsymbol{p}_s)}, \tag{3}$$

which is evidently independent of the utility level. All conventional price indices that can be derived from economic theory satisfy this property.

2.2 The Nonhomothetic Case: Preferences

Our framework extends the indirect utility (2) to allow for nonhomothetic behavior. To this end, we characterize preferences by an indirect utility function as in Boppart (2014),

$$V(e, \boldsymbol{p}) = \frac{1}{\varepsilon}\left[\left(\frac{e}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right], \tag{4}$$

where $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ are linearly homogeneous functions of prices and the parameters satisfy $\varepsilon, \gamma \quad (0, 1)$ and $\nu > 0$. This utility function belongs to the class of PIGL preferences defined by Muellbauer (1975, 1976) and more generally to the class of "intertemporally aggregable" preferences defined by Alder, Boppart and Müller (2022). Despite being nonhomothetic, these preferences consistently aggregate across individual-level consumption expenditures. Aggregate expenditure shares in this case correspond to a representative expenditure level which is independent of prices and given by the average expenditure level multiplied by a simple inequality measure.

To gain understanding and intuition of Equation (4), it is convenient to think of $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ as the expenditure functions of some homothetic consumption bundles. We refer to these bundles as "goods" or "baskets". The parameter ε controls the degree of nonhomotheticity between the D and B baskets: the expenditure elasticity of demand for the D basket is $1-\varepsilon$, which is less than 1 under the restrictions on ε. The D basket therefore covers necessity needs and B conversely covers luxury needs. In the limit case $\varepsilon \to 0$, the expenditure elasticity is 1 and we obtain homothetic preferences. Comparing Equations (2) and (4), we also obtain homothetic preferences for $\varepsilon = 0$ whenever $B(\boldsymbol{p}) = D(\boldsymbol{p})$ or in the limit case $\nu \to 0$. The parameter ν is a scale parameter that controls the level of demand for the D basket and γ controls the nonconstant elasticity of substitution between the B and D baskets. By Boppart (2014, Lemma 3), this elasticity of substitution is given by $1-\gamma-\frac{w_D}{w_B}(\gamma-\varepsilon)$, where w_D and w_B are the expenditure shares allocated to the D and B baskets.

In general, there is nothing restricting an individual good j from occurring in both the B and the D baskets. If there is overlap between the sets of goods within B and D, the allocations to the B and D goods are not directly observable and we obtain what Blundell and Robin (2000) call "latent separability". Latent separability is equivalent to quasi separability, but in the latent goods B and D rather than in purchased goods, with quasi separability as the special case when there is no overlap between B and D. Two-stage budgeting is still valid under latent separability, meaning that the consumer's allocation problem can be viewed in two stages where consumers first allocate expenditures between the B and D baskets and then, conditional on this first-stage decision, allocate expenditures across individual goods within B and D. Applying Roy's identity, the expenditure shares w_D and w_B allocated to the D and B baskets in the first stage are therefore given by

$$w_D = \nu \left(\frac{B(\boldsymbol{p})}{e}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} \tag{5}$$

and

$$w_B = 1 - \nu \left(\frac{B(\boldsymbol{p})}{e}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}. \tag{6}$$

Similarly, the shares w_j^D and w_j^B of total D and B expenditures allocated to individual good j are given by

$$w_j^D = p_j \frac{D_j(\boldsymbol{p})}{D(\boldsymbol{p})} \qquad \text{and} \qquad w_j^B = p_j \frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})}, \tag{7}$$

where D_j and B_j denote the partial derivatives of D and B with respect to p_j. Equations (5) to (7) imply an expenditure share w_j of good j in total expenditures of the form

$$w_j = p_j \left[\frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} + \left(\frac{D_j(\boldsymbol{p})}{D(\boldsymbol{p})} - \frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} \right) \nu \left(\frac{B(\boldsymbol{p})}{e}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} \right]. \tag{8}$$

Therefore, nonhomotheticity between B and D also creates nonhomothetic behavior across individual goods, with a good j being a necessity if $D_j/D > B_j/B$ and a luxury vice versa. Aggregating over any measure N of consumers indexed by h, the aggregate expenditure share $\overline{w}_j$ of good j across these consumers is

$$\overline{w}_j = p_j \left[\frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} + \left(\frac{D_j(\boldsymbol{p})}{D(\boldsymbol{p})} - \frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} \right) \nu \left(\frac{B(\boldsymbol{p})}{\bar{e}} \right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})} \right)^{\gamma} \kappa \right], \tag{9}$$

where $\bar{e} \equiv \frac{1}{N} \int_0^N e_h \, dh$ is the average expenditure level and κ is a scale-invariant inequality measure defined by

$$\kappa \equiv \frac{1}{N} \int_0^N \left(\frac{e_h}{\bar{e}} \right)^{1-\varepsilon} dh. \tag{10}$$

Aggregate shares $\overline{w}_B$ and $\overline{w}_D$ of the B and D baskets are defined similarly. See Alder, Boppart and Müller (2022, Proposition 2) for a derivation of Equations (9) and (10). The representative agent in Muellbauer's (1975, 1976) sense is the expenditure level e^{RA} that induces the aggregate expenditure share. By Equation (9), this expenditure level is given by $e^{RA} \equiv \bar{e}\kappa^{-1/\varepsilon}$.

2.3 The Nonhomothetic Case: Price Index

The indirect utility function (4) allows us to extend the homothetic cost-of-living index (3). Unlike the homothetic case, the index now depends on a base-period standard of living, represented by the utility level u in the Konüs definition (1). Because preferences are nonhomothetic over the B and D baskets, with an expenditure elasticity of demand for the D basket always less than 1, this utility level is fully captured by w_{Ds}, the base-period expenditure share of the D good. For the remainder of the paper, let

$$L(x, y) = \begin{cases} \dfrac{x - y}{\ln x - \ln y} & \text{if } x = y, \\ x & \text{if } x = y, \end{cases}$$

denote the logarithmic mean (Carlson, 1972). Moreover, recalling from Equation (3) that the cost-of-living index of homothetic preferences between some period t and base period s is the ratio of the corresponding expenditure functions over the same periods, we denote the price indices of the B and D baskets by $P_{Bt} \equiv B(\boldsymbol{p}_t)/B(\boldsymbol{p}_s)$ and $P_{Dt} \equiv D(\boldsymbol{p}_t)/D(\boldsymbol{p}_s)$. The following result then shows that the cost-of-living index corresponding to Equation (4) is a function of w_{Ds}, P_{Bt} and P_{Dt} and the two parameters ε and γ.

Proposition 1 (PIGL cost-of-living index)**.** *If preferences are of the PIGL form (4) and the base-period expenditure share w_{Ds} allocated to the D basket is given, the Konüs cost-of-living index is*

$$P_t^{PIGL} = P_{Dt}^{\frac{\gamma\phi_t}{\varepsilon}} P_{Bt}^{1-\frac{\gamma\phi_t}{\varepsilon}} \qquad \textit{with} \qquad \phi_t \equiv \frac{L(\psi_{Dt},\ \psi_{Ds})}{L(\psi_{Dt},\ \psi_{Ds})\ +\ L(\psi_{Bt},\ \psi_{Bs})}, \tag{11}$$

where

$$\psi_{Bt} \equiv \left(1 - \frac{\varepsilon w_{Ds}}{\gamma}\right)\left(\frac{P_{Bt}}{\widetilde{P}_t}\right)^{\gamma} \qquad \textit{and} \qquad \psi_{Dt} \equiv \frac{\varepsilon w_{Ds}}{\gamma}\left(\frac{P_{Dt}}{\widetilde{P}_t}\right)^{\gamma} \tag{12}$$

are shares of a CES-type aggregator $\widetilde{P}_t$ defined by

$$\widetilde{P}_t \equiv \left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma}\right) P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma} P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}}. \tag{13}$$

The aggregate cost-of-living index over any measure N of consumers is given identically using their average expenditure share $\overline{w}_{Ds}$ in ϕ_t.

Sketch proof (full proof in Appendix A.1)*.* Set the reference utility to that of the base period expenditure level, $u \equiv V(e_s, \boldsymbol{p}_s)$. It is then possible to write the period-t expenditure function corresponding to Equation (4) as $c(u, \boldsymbol{p}_t) = c(u, \boldsymbol{p}_s)\widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}$. Since $\widetilde{P}_t$ is of a CES form, it can be rewritten as a Sato-Vartia index with weights $1 - \phi_t$ and ϕ_t on P_{Bt} and P_{Dt}, respectively. The result then follows from the Konüs definition (1). □

Proposition 1 shows that the PIGL cost-of-living index can be written as something akin to a Sato-Vartia index over the B and D baskets. Unlike the homothetic case, however, the ϕ_t in the weights of the two subindices varies across the expenditure distribution. Richer consumers spend a smaller share w_D on the D basket, which reduces the weights ψ_{Dt} and, subsequently, ϕ_t. In other words, because richer consumers allocate a smaller share to the D basket, the corresponding price index P_{Dt} is weighted less heavily when determining the overall change in the cost of living. The weights ψ_{Dt} and ψ_{Bt} are not directly observable but are readily computed given price indices P_{Bt} and P_{Dt}, an expenditure share w_{Ds}, and parameter values for ε and γ. In Appendix A.1, we show that $w_{Ds}(P_{Dt}\,/\,\widetilde{P}_t)^{\gamma}$ is the expenditure share of the D basket at period-t prices that prevails at the same utility level as w_{Ds}. Therefore, the weights ψ_{Dt} and ψ_{Bt} ensure that the consumer remains on the same indifference curve as in the base period.

Two potential caveats to Proposition 1 are that the underlying preferences are identical across consumers with the same expenditure level and that expenditure shares change monotonically in the level of expenditure. Redding and Weinstein (2020) emphasize accounting for taste heterogeneity in cost-of-living indices while Banks, Blundell and Lewbel (1997) highlight the importance of allowing for hump-shaped expenditure shares to match microeconomic data. In Appendix A.4 we show that it is straightforward to incorporate time- and household-specific tastes between the B and D baskets into the indirect utility function (4) and that this leaves Proposition 1 unaffected. In Appendix A.5 we discuss a generalization that allows for hump-shaped expenditure shares. This generalization works well for household-level indices but requires more stringent conditions for aggregate price indices to have the same form.

2.4 Generalized Superlative Indices

With Proposition 1 at hand, it is straightforward to generalize standard homothetic indices to the nonhomothetic PIGL case: just plug in two homothetic indices for P_{Bt} and P_{Dt} in Equation (11). To emphasize the importance of Proposition 1, we present generalizations of two classes of indices: Diewert's (1976) ***quadratic-mean-of-order-r*** class, which consists of all indices of the form

$$P_t = \sqrt{\left\{\sum_{j \in J} w_{js} \left(\frac{p_{jt}}{p_{js}}\right)^{\frac{r}{2}}\right\}^{\frac{2}{r}} \left\{\sum_{j \in J} w_{jt} \left(\frac{p_{jt}}{p_{js}}\right)^{-\frac{r}{2}}\right\}^{-\frac{2}{r}}}, \qquad r > 0, \tag{14}$$

and Barnett and Choi's (2008) ***Theil-Sato*** class, which is defined as

$$P_t = \prod_{j \in J} \left(\frac{p_{jt}}{p_{js}}\right)^{\delta_{jt}}, \qquad \delta_{jt} \equiv \frac{m(w_{jt}, w_{js})}{\sum_{i \in J} m(w_{it}, w_{is})}, \tag{15}$$

where $m(x, y)$ is a ***symmetric mean*** of two variables, a function class that includes all linearly homogenous functions satisfying $\min\{x, y\} \leq m(x, y) = m(y, x) \leq \max\{x, y\}$. These index classes include several of the most well-known price index formulas. Equation (14) incorporates Fisher's (1922) ideal index ($r = 2$), the arithmetic Walsh (1901) index ($r = 1$) and, as a limit case, the Törnqvist (1936) index ($r \to 0$). Equation (15) nests the Törnqvist index (arithmetic mean, $m(x, y) = (x + y)/2$), the geometric Walsh (1901) index (geometric mean, $m(x, y) = \sqrt{xy}$), the Sato (1976)-Vartia (1976) index (logarithmic mean, $m(x, y) = L(x, y)$), and the Theil (1973) index ($m(x, y) = \sqrt[3]{xy(x + y)/2}$). While we could choose any underlying homothetic price indices P_{Dt} and P_{Bt}, these two classes encompass all currently known ***superlative*** price indices (Diewert, 1976). That is, they are exact cost-of-living indices for some homothetic expenditure functions and are second-order approximations of any other homothetic price index.[5] Therefore, even if the indices corresponding to the true expenditure functions $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ have some other forms than those in Equations (14) and (15), we should still be able to reasonably approximate their corresponding price indices under this specific parameterization. The nonhomothetic generalization of these indices under Proposition 1 is presented below.

[5] This definition differs from Diewert's original definition but is shown by Barnett and Choi (2008, Theorem 1) to be equivalent.

Corollary 1 (Generalized superlative indices)**.** *If preferences are of the PIGL form (4), the base-period expenditure share w_{Ds} allocated to the D basket is given, and $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ are expenditure functions with price indices of the form (14) or (15), the Konüs cost-of-living index is*

$$P_t = \prod_{j \quad J} \left(\frac{p_{jt}}{p_{js}} \right)^{\chi_{jt}}, \tag{16}$$

where

$$\chi_{jt} \equiv \frac{\gamma \phi_t}{\varepsilon} \, \delta^D_{jt} + \left(1 - \frac{\gamma \phi_t}{\varepsilon} \right) \delta^B_{jt}, \tag{17}$$

with ϕ_t as in Proposition 1. The weights δ^C_{jt}, $j \quad J$, $C \quad \{B, D\}$, are given by

$$\delta^C_{jt} = \frac{1}{2} \left[\frac{\widetilde{w}^C_{Ljt}}{\sum_i \widetilde{w}^C_{Lit}} + \frac{\widetilde{w}^C_{Pjt}}{\sum_i \widetilde{w}^C_{Pit}} \right] \quad \text{or} \quad \delta^C_{jt} = \frac{m(w^C_{jt}, w^C_{js})}{\sum_i m(w^C_{it}, w^C_{is})} \tag{18}$$

if P_{Ct} is as in Equation (14) or (15), respectively. In the former case,

$$\widetilde{w}^C_{Ljt} \equiv w^C_{js} \, L\left(\left(\frac{p_{jt}}{p_{js}} \right)^{\frac{r}{2}}, \left(P^C_{Lt} \right)^{\frac{r}{2}} \right) \quad \text{and} \quad \widetilde{w}^C_{Pjt} \equiv w^C_{jt} \, L\left(\left(\frac{p_{jt}}{p_{js}} \right)^{-\frac{r}{2}}, \left(P^C_{Pt} \right)^{-\frac{r}{2}} \right),$$

where $P^C_{Lt} = \left[\sum_j w^C_{js} \left(\frac{p_{jt}}{p_{js}} \right)^{\frac{r}{2}} \right]^{\frac{2}{r}}$ and $P^C_{Pt} = \left[\sum_j w^C_{jt} \left(\frac{p_{jt}}{p_{js}} \right)^{-\frac{r}{2}} \right]^{-\frac{2}{r}}$. The aggregate cost-of-living index over any measure N of consumers is given identically using their average expenditure shares in χ_{jt}.

Proof. In Appendix A.2. □

In Corollary 1, we have rewritten the *quardratic-mean-of-order-r* class on a geometric-mean form following Balk (2004) to highlight the intuitive generalization of the homothetic superlative indices that we obtain. In doing so, we denote the weights by L and P to capture the fact that these reduce to standard Laspeyres and Paasche weights when $r = 2$. The resulting cost-of-living index is a weighted geometric average of individual price changes with the index weights (17) of the following structure:

$$\begin{matrix}\text{Weight} \\ \text{on } j\end{matrix} = \begin{matrix}\text{Weight} \\ \text{on } D\end{matrix} \times \begin{matrix}\text{Weight} \\ \text{on } j \\ \text{within } D\end{matrix} + \begin{matrix}\text{Weight} \\ \text{on } B\end{matrix} \times \begin{matrix}\text{Weight} \\ \text{on } j \\ \text{within } B\end{matrix}.$$

The weights on j within B and D, given by Equation (18), are standard homothetic weights and affect all consumers similarly. The weights on D and B are the same as in Proposition 1. Therefore, the overall weights χ_{jt} vary across the base-period expenditure distribution in a similar way as before. If $B(\boldsymbol{p}) = D(\boldsymbol{p})$, we get that $\delta^B_{jt} = \delta^D_{jt}$ for all j and the generalized superlative indices immediately collapse to the homothetic indices in Equations (14) and (15).

At first sight, Corollary 1 looks similar to the methodology used in much of the literature concerned with inflation inequality, whereby homothetic price indices are computed for different income groups separately. In particular, papers like Broda and Romalis (2009), Jaravel (2019), Argente and Lee (2021), and Klick and Stockburger (2021) construct homothetic price indices of the geometric-mean form $\ln P_t = \sum_j \delta_{jt} \ln(p_{jt}/p_{js})$, where δ_{jt} are weights computed separately for each income group considered based on observed consumer behavior. This generates heterogeneous weights across the income distribution. The method therefore mimics an overall geometric-mean price index with income specific weights, which is exactly what we also have in Corollary 1. An important distinction between the two, however, is that Corollary 1 provides a cost-of-living index that guarantees that a fixed standard of living across time periods. This contrasts with the approach in previous papers: by relying on changes in *observed* consumer behavior over time, the constructed index weights in these papers will also capture changes in the standard of living over time, which may bias the price indices.

3 Empirical Implementation

To compute the generalized superlative indices in practice, we need total expenditure shares between the B and D baskets and the expenditure shares within each basket. Yet, if individual goods occur in both the B and the D baskets, these across and within expenditure shares are unobserved in the data. The only feasible approach then is to parameterize $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$, estimate the demand system associated with the expenditure share equations (8) via GMM, and infer these shares from the estimated model. This methodology, however, suffers from the usual drawbacks of nonlinear demand system estimation. In particular, for standard parameterizations the number of parameters to estimate quickly grows out of proportion as we increase the number of goods considered.[6] The nonlinear nature of the demand system also implies that there is no guarantee that the GMM estimator converges to the actual global minimum of the GMM objective function. The latter could in principle be solved by a grid search, but this only exacerbates the curse of dimensionality further. Estimating more than a few goods is therefore generally infeasible. These issues, however, are fully circumvented within our framework when a simple assumption on the structure of the demand system is met.

Assumption 1. Preferences are quasi separable into $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$. $\triangleleft$

Under Assumption 1, the price of an individual good occurs in either $B(\boldsymbol{p})$ or $D(\boldsymbol{p})$, but not in both. Since the D basket captures necessity needs and B basket luxury needs, it follows that preferences are also quasi separable into necessities and luxuries. The assumption is therefore easily implemented empirically by allocating luxuries to B and necessities to D.

The immediate consequence of Assumption 1 is that across and within expenditure shares become observable in the data. Summing the total expenditure shares w_j (which are always observable) over goods in D gives the across share w_D. Within shares are then obtained as $w_j^D = w_j/w_D$. The same applies for the B basket. This knowledge is enough to compute price indices P_{Bt} and P_{Dt} for the B and D baskets using Equation (14) or (15). By Proposition 1, the only additional components needed to compute the generalized superlative indices are then the two parameters ε and γ. Using Equations (3) and (5), we may write the period-t expenditure share on the D

[6] As an illustration, suppose we have n goods and parameterize $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ by the linearly homogeneous translog expenditure function of Christensen, Jorgenson and Lau (1975), for which the Törnqvist index is an exact cost-of-living index (Diewert, 1976). The PIGL demand system considered here then requires the estimation of $n(n+1)+3$ independent parameters, a number which grows exponentially with n.

good as

$$w_{Dt} = \tilde{\nu} \left(\frac{P_{Bt}}{e_t} \right)^{\varepsilon} \left(\frac{P_{Dt}}{P_{Bt}} \right)^{\gamma}, \tag{19}$$

where $\tilde{\nu} \equiv \nu B(\boldsymbol{p}_s)^{\varepsilon-\gamma} D(\boldsymbol{p}_s)^{\gamma}$ is a scale parameter. Since w_{Dt}, e_t, P_{Bt} and P_{Dt} are all known, estimating ε and γ from (19) is easily carried out using either linear (by taking logs of (19)) or nonlinear estimation methods. We summarize this empirical approach by the following proposition:

Proposition 2 (Tractable demand system estimation)**.** *Under Assumption 1, across and within group expenditure shares are observable in the data and computing the generalized superlative indices* (16) *only requires estimation of two parameters, ε and γ, from the single expenditure share equation* (19)*.*

At first sight, Assumption 1 may seem to be at odds with the nonhomothetic generalization of the superlative indices: Corollary 1 reduces to the standard homothetic case when $B(\boldsymbol{p}) = D(\boldsymbol{p})$, which Assumption 1 excludes by construction. However, the quasi separability assumption still nests homothetic preferences. In particular, as $\varepsilon \to 0$ and $\gamma \to 0$, we obtain Cobb-Douglas preferences with an indirect utility function given by $V(e, \boldsymbol{p}) = \ln \left[\frac{e}{B(\boldsymbol{p})^{1-\nu} D(\boldsymbol{p})^{\nu}} \right]$. The corresponding price index is then $P_t = P_{Bt}^{1-\nu} P_{Dt}^{\nu}$, where $\nu = w_D$ is the homothetic and time-invariant expenditure share on D. Thus, if preferences truly are homothetic, we still expect Proposition 2 to yield a homothetic price index. This index is approximately equal to the corresponding superlative index when $B(\boldsymbol{p}) = D(\boldsymbol{p})$, by virtue of superlative indices being second-order approximations of any other homothetic index. This highlights that using the generalized superlative indices under Assumption 1 should at least (approximately) be weakly better than using the standard homothetic indices. Since nonhomothetic preferences is the empirically relevant case, we do not expect the "approximately" part to matter much, and the empirical application below confirms this. For cases where it nevertheless might be of importance, it turns out that a special case exists where Assumption 1 exactly nests the corresponding homothetic index when $B(\boldsymbol{p}) = D(\boldsymbol{p})$: the Törnqvist index.

Proposition 3 (Homothetic Törnqvist index under quasi separability)**.** *Suppose that preferences are of the homothetic Cobb-Douglas form, $V(e, \boldsymbol{p}) = \ln \left[\frac{e}{B(\boldsymbol{p})^{1-\nu} D(\boldsymbol{p})^{\nu}} \right]$, and that $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ are such that their corresponding price indices P_{Bt} and P_{Dt} are Törnqvist indices. The cost-of-living index under Assumption 1 is then the standard Törnqvist index:*

$$P_t = \prod_{j \ J} \left(\frac{p_{jt}}{p_{js}} \right)^{\delta_{jt}}, \qquad \text{where} \qquad \delta_{jt} = \frac{w_{js} + w_{jt}}{2}.$$

Proof. In Appendix A.3. □

4 Data and Estimation

We implement the tractable demand system estimation described in the previous section using consumption and price data from two sources. Household consumption is taken from the interview component of the Consumer Expenditure Survey (CEX) and price data are taken from the product-level Consumer Price Index (CPI) series for all urban consumers. Both are provided by the US Bureau of Labor Statistics (BLS). The CEX interview survey is a quarterly rotating panel of households who are representative of the US population. New households are sampled every month and each household is tracked for up to four consecutive quarters. The survey covers around 95 percent of total household consumption and contains additional information on annual income, socioeconomic characteristics and other background characteristics like ownership of a car. The survey has been continuously conducted since 1980, though we focus on the years 1995 to 2020 to ensure consistency across waves and to match the availability of the CPI subindices.

As is standard in the literature, we select a sample of respondents between the ages of 25 and 65 who report strictly positive income. To avoid issues with seasonality, we aggregate expenditures to annual levels and, consequently, drop households that do not respond to all four quarterly interviews. To account for differences in household size, we also divide household income and expenditures by the number of adult equivalents in the household using the equivalence scale of the US Census Bureau (see Fox and Burns, 2021). The final dataset on expenditures consists of approximately 3,000 households per year.

We aggregate nondurable consumption expenditures into a rather coarse set of consumption goods categories as this allows us to compare the empirical approach in Proposition 2 with a full demand system estimation. All in all, we consider twenty-one expenditure categories using the hierarchical groupings defined by the BLS. We broadly follow Hobijn and Lagakos (2005) and construct prices for these categories by matching them with individual CPI series. Table B.1 in Appendix B lists the CEX categories and shows their mapping to the CPI item codes.

4.1 Classification of Goods Into Luxuries and Necessities

To utilize the tractable demand system estimation in Proposition 2, we impose Assumption 1 by allocating luxuries to B and necessities to D. The classification into B and D is implemented by investigating slopes of the budget-share Engel curves: if the Engel curve of a good decreases as expenditures increase, it is a necessity. Conversely, a good is a luxury if its Engel curve increases with increasing expenditures. We split households into expenditure deciles and, for each good j, run a household-level regression of the expenditure share w_{jh} on the expenditure decile d_h of household h:

$$w_{jh} = \alpha_j + \beta_j \mathsf{d}_h + \epsilon_{jh}.$$

If $\beta_j > 0$, we allocate the good to the B basket, otherwise we allocate it to the D basket. Figure 1 shows the Engel curves by expenditure decile together with the resulting classification from the regressions. The resulting classification is intuitive and for comparable groups the necessity/luxury split is highly similar to those constructed in similar analyses using CEX data (see for instance Wachter and Yogo, 2010, and Orchard, 2022), thus suggesting that this simple approach works well on our coarse set of goods. Table B.2 in Appendix B lists the β_j coefficient estimates along with robustness specifications including regressions on the level and

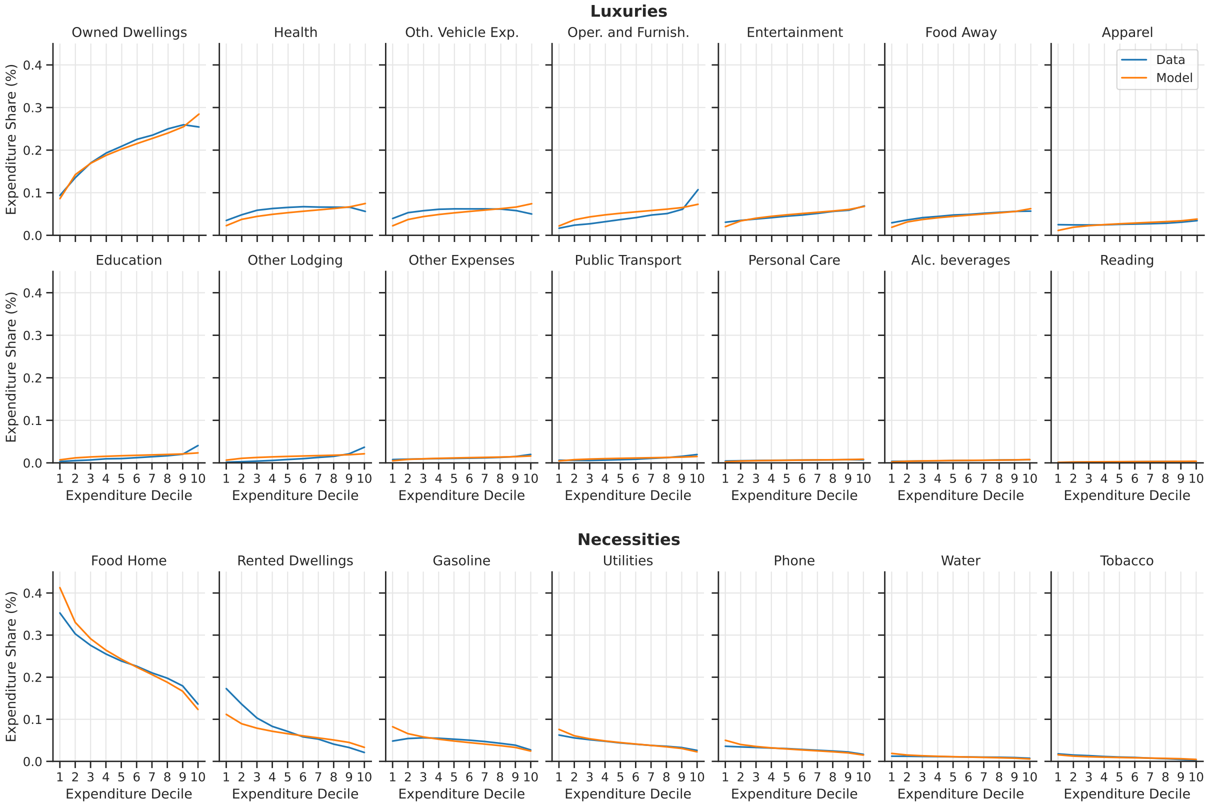

Figure 1. Empirical and model implied Engel curves.

Notes. The figure shows the empirical and model implied expenditure shares by expenditure group and expenditure decile averaged over all years. The model implied expenditure shares correspond to those under the assumption of quasi separability and the Sato-Vartia specification. They are calculated by first taking the model implied expenditure shares on the B and D goods and then use the empirical expenditures shares of all households to obtain shares within these groups.

log of expenditures instead of the decile group and additional controls. The estimates of the β_j coefficients are always significantly different form zero and their signs never change across robustness specifications.

4.2 Tractable Demand System Estimation

We estimate the preference parameters ε and γ from Equation (19) using a nonlinear GMM estimator on the CEX data. In doing so, we make explicit corrections for two potential issues.

First, it is well known that infrequently bought items, like clothing and transportation goods and services, create a measurement error in the observed level of expenditures. Although we alleviate much of this concern by excluding durable goods and by aggregating expenditures to an annual level, we follow the literature (Blundell, Pashardes and Weber, 1993; Banks, Blundell and Lewbel, 1997) and control for this endogeneity bias by using household income as an instrument for household consumption expenditure.

Second, an indirect utility specification like the PIGL requires additional attention with respect to the regularity conditions for utility maximization. Specifically, we need to certify that the parameter estimates yield a symmetric and negative semidefinite Slutsky matrix. Under the indirect utility function (4) and Assumption 1, it follows from Boppart (2014, Lemma 1) that a necessary and sufficient condition for household h to satisfy the Slutsky restrictions in period t is $\tilde{\nu}\left(\frac{P_{Bt}}{e_{ht}}\right)^{\varepsilon}\left(\frac{P_{Dt}}{P_{Bt}}\right)^{\gamma} \leq (1-\gamma)/(1-\varepsilon)$. We enforce this constraint by augmenting the GMM estimation with a standard penalty method, in which we add prohibitive penalties to the GMM objective function outside of the feasible region of parameter values. Consequently, the reported parameter estimates below satisfy the Slutsky restrictions for all observations in the sample.

To gauge the sensitivity to different choices of underlying superlative price indices, we estimate ε and γ for six different choices of P_{Bt} and P_{Dt}. These choices correspond to the indices listed in Section 2.4: the Sato-Vartia, the Törnqvist, the Walsh (geometric and arithmetic), the Theil, and the Fisher indices. This robustness check is instructive since there is generally no guarantee that superlative indices are numerically similar, despite being second-order approximations of each other (see for instance Hill, 2006). For the estimation exercise, we compute these indices on a monthly frequency using aggregate expenditure shares of all households in the CEX. Since individual household's expenditures are aggregated to a twelve-month period, we also aggregate the price levels they face by taking the expenditure weighted average over the months each household is in the sample.

The estimated parameters for the six cases are reported in Table 1. All parameters are significantly different from zero at conventional significance levels and the fact that $\varepsilon > 0$ and $\tilde{\nu} > 0$ directly rejects homotheticity. Reassuringly, the choice of price indices for the B and D baskets turns out to be completely inconsequential as all specifications yield close to identical estimates.[7] Moreover, Alder, Boppart and Müller (2022, Proposition 3) show that a sufficient condition for predicted expenditure shares to remain globally nonnegative is $0 < \gamma \leq \varepsilon < 1$. This condition is also met in our estimation, though we do not impose the constraint explicitly. Other preference specifications, like the Almost Ideal Demand System, typically violate expenditure share nonnegativity for sufficiently large expenditures levels.

To get an idea of how well the estimated model matches the data, we compute budget-share Engel curves using the parameter estimates for the Sato-Vartia specification and plot these against their empirical counterparts in Figure 1. For a good j in basket $C \in \{B, D\}$, we construct model expenditure shares as the product of the model-implied across-share w_C and observed within-shares w_j^C. The across-share is computed from Equation (19) at the representative level of expenditures within each expenditure decile. Since $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ are homothetic, the within-shares are given empirically by the average within-shares $\overline{w}_j^B$ and $\overline{w}_j^D$ across ***all*** households. Although we of course do not capture ***all*** nonhomothetic consumption patterns,[8] the resulting model-implied Engel curves are nonconstant and exhibit reasonably similar patterns as in the data. This contrasts to the constant Engel curves induced by homothetic preferences, which is the underlying preference structure assumed in all other conventional price indices. Using

[7] The estimates in Table 1 weigh observations by their CEX sampling weights but are robust to not using these weights.

[8] Figure B.1 in Appendix B shows how expenditure shares in the data vary *within* the luxury and necessity baskets across expenditure deciles once we impose Assumption 1, which highlights the nonhomotheticity ignored under quasi separability.

Table 1. GMM estimates of the PIGL parameters under weak separability.

	Sato-Vartia	Törnqvist	Geometric Walsh	Theil	Fisher's Ideal	Arithmetic Walsh
ε	0.677	0.677	0.677	0.677	0.677	0.677
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
γ	0.211	0.211	0.211	0.211	0.211	0.211
	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)
$\widetilde{\nu}$	327.271	327.437	327.173	327.273	324.273	324.600
	(13.358)	(13.365)	(13.354)	(13.358)	(13.217)	(13.233)
Observations	74,372	74,372	74,372	74,372	74,372	74,372
RMSE	0.1487	0.1486	0.1487	0.1487	0.1486	0.1486

Notes. Robust standard errors in parentheses. "RMSE" refers to the root-mean-square error of the expenditure share on the D good: $\sqrt{\sum_h (w_{Dh} - \widehat{w}_{Dh})^2 / N}$. Observation are weighted by their CEX sampling weights.

this as our point of departure, Figure 1 underscores that the assumption of quasi separability between the B and D baskets is not a strong restriction for our price index purposes.

5 Cost-of-Living and Inflation Inequality

The estimation in the previous section suggests that the results are insensitive to the choice of generalized superlative index.[9] For the remainder of the paper, we therefore focus on the nonhomothetic generalization of the Sato-Vartia index (which we creatively refer to as the Generalized Sato-Vartia price index from now on). Since the Sato-Vartia index is the Konüs index corresponding to the canonical CES expenditure function (see Sato, 1976), this choice implies that we are investigating a generalization of homothetic CES preferences. The prominence of CES preferences within macroeconomics and international trade makes this a case of particular interest. The main reason for selecting the generalization of the Sato-Vartia index, however, is that a parameterization of $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ as CES aggregates contains many fewer parameters than the parameterizations that induce, for instance, the Fisher or the Törnqvist indices. This allows us to compare the Generalized Sato-Vartia price index later on to a parsimonious demand system estimation of these preferences when we do not invoke the quasi separability assumption.

5.1 Main Results

Figure 2 shows the evolution of the Generalized Sato-Vartia price index from 1995 to 2020. We set the base period to 1995 and, in contrast to Section 4.2, use annual indices for P_{Bt} and P_{Dt}.[10] Even though the generalized superlative price indices allow for characterizations of the entire distribution of indices, here we focus on expenditure deciles for ease of exposition. Figures B.4 and B.5 in Appendix B show the full price index and inflation distributions.

[9] Figures B.7 and B.8 in Appendix B show the main results in this section for all choices considered in Section 4.2, which confirms that this is indeed the case.

[10] Figure B.2 in Appendix B shows that the choice of base period does not affect our results.

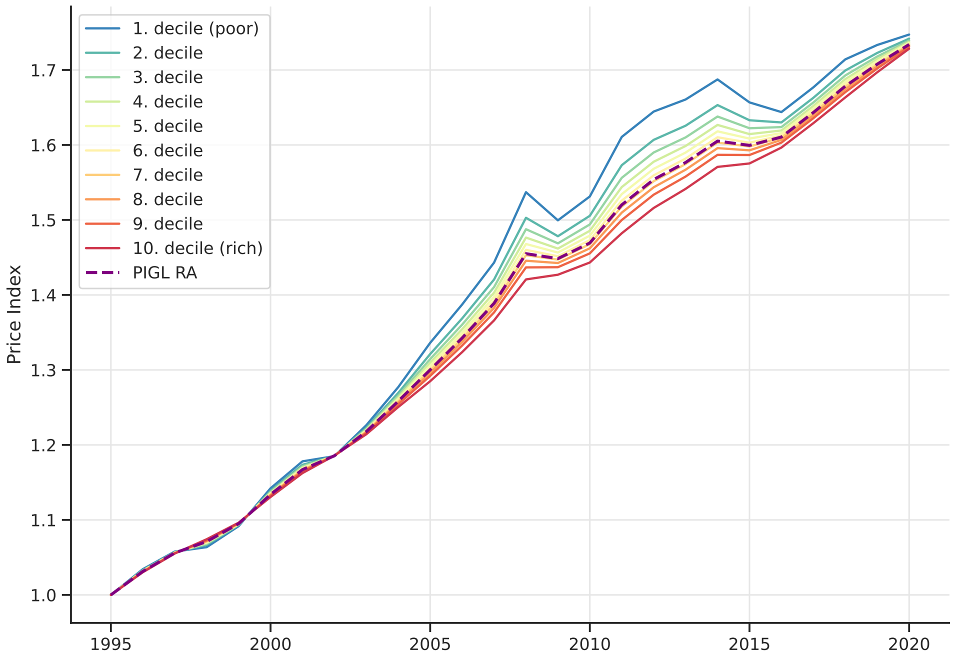

Figure 2. Generalized Sato-Vartia price index under quasi separability by expenditure decile.

Notes. The price index for each expenditure decile is calculated as the price index of the PIGL representative agent over households within each respective decile in the base year. "PIGL RA" stands for the PIGL representative agent over *all* households.

Figure 2 corroborates two findings from the literature: inflation rates vary across households and poorer households have experienced a larger increase in the cost of living than richer households over the last quarter century. It is noteworthy, though, that the cumulative differences are small. Table 2 makes this point clear: the mean annual inflation rate of the poorest ten percent is only 0.06 percentage points higher than that of the richest ten percent over the full sample period. Consequently, the changes in the cost of living over the 26 years under study do not diverge dramatically between households.

The small differences in the cost of living by 2020 is striking given the substantial heterogeneity observed in subperiods of the sample. For instance, if we zoom in on the years 2004 to 2015, the annual change in the cost of living for the poorest ten percent are on average 0.37 percentage points higher than the change for the richest ten percent. Jaravel (2019) and Argente and Lee (2021) focus on the same years, the former using a CEX-CPI dataset similar to ours and the latter using scanner data for the retail sector, and both find results close to ours. To put this difference in perspective, the Boskin Commission Report estimated the total bias in the aggregate US CPI to be 1.1 percentage points (Boskin *et al.*, 1996). Of these, substitution biases alone account for 0.4 percentage points. The difference we find here is therefore substantial when compared to previously estimated biases in aggregate price indices.

That the differences in the change in the cost of living varies substantially across subperiods is also visible from the implied annual inflation rates, which we plot in Figure 3. In several years, the range of inflation rates across the expenditure distribution exceeds 2 percentage points.

Table 2. Inflation rate levels and dispersion over time across the consumption distribution.

	1996–2020				2004–2015			
	Level		Dispersion		Level		Dispersion	
Decile	Mean (%)	Relative to top decile (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)	Mean (%)	Relative to top decile (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)
1	2.28	0.06	2.14	2.51	2.57	0.37	2.71	2.38
2	2.26	0.04	1.76	2.06	2.46	0.26	2.25	1.98
3	2.25	0.03	1.59	1.86	2.41	0.21	2.05	1.79
4	2.24	0.03	1.46	1.71	2.38	0.18	1.89	1.66
5	2.24	0.02	1.36	1.60	2.35	0.15	1.78	1.56
6	2.24	0.02	1.28	1.50	2.33	0.13	1.67	1.46
7	2.23	0.02	1.20	1.40	2.30	0.10	1.57	1.38
8	2.23	0.01	1.12	1.31	2.28	0.08	1.47	1.29
9	2.22	0.01	1.02	1.20	2.25	0.05	1.35	1.18
10	2.22	0.00	0.85	1.00	2.20	0.00	1.14	1.00

Notes. Arithmetic mean and standard deviation of annual inflation over the time periods 2004–2015 and 1996–2020. Under inflation rate levels, the "Relative to top decile" columns show the percentage point difference in the average annual inflation rate to that of the tenth consumption decile. Under inflation rate dispersion, the same columns show the standard deviation of annual inflation as a multiple of that of the tenth consumption decile.

Moreover, poor households experience higher inflation than rich households in periods of high inflation but also experience *lower* inflation than the rich in periods of *low* inflation. Table 2 outlines these differences in inflation rate volatility and shows that the standard deviation of inflation is 2.14 for the poorest and 0.85 for the richest. The poorest households therefore experience a 2.5 times *more volatile* inflation rate than the richest households.

In sum, despite the fact that the overall change in the cost of living has not diverged dramatically between groups, there is a considerable difference in the volatility of inflation rates. This in turn generates significant differences in cost-of-living changes during subperiods of our sample. These findings immediately raise the question of *why* poorer households face this higher cost-of-living volatility, and this is what we turn to next.

5.2 What Drives Inflation and the Cost of Living for Different Households?

The nonhomothetic nature of the underlying preferences that we rely on suggests two main channels at work in Figures 2 and 3. On the one hand, poorer households allocate a larger share of consumption expenditures on necessity items like food and energy, as highlighted by the Engel curves in Figure 1. This implies that poorer households are more exposed to price changes of these product categories than richer households. Conversely, rich households put relatively more weight on price changes of luxury goods. Therefore, if prices of necessities fluctuate more than those of luxuries, we should expect inflation for poor households to also fluctuate more. On the other hand, the direct impact of a price change on the overall cost of living may be offset by substitution towards relatively cheaper expenditure categories. If this substitution effect varies across the expenditure distribution, this too contributes to the different changes in the cost of

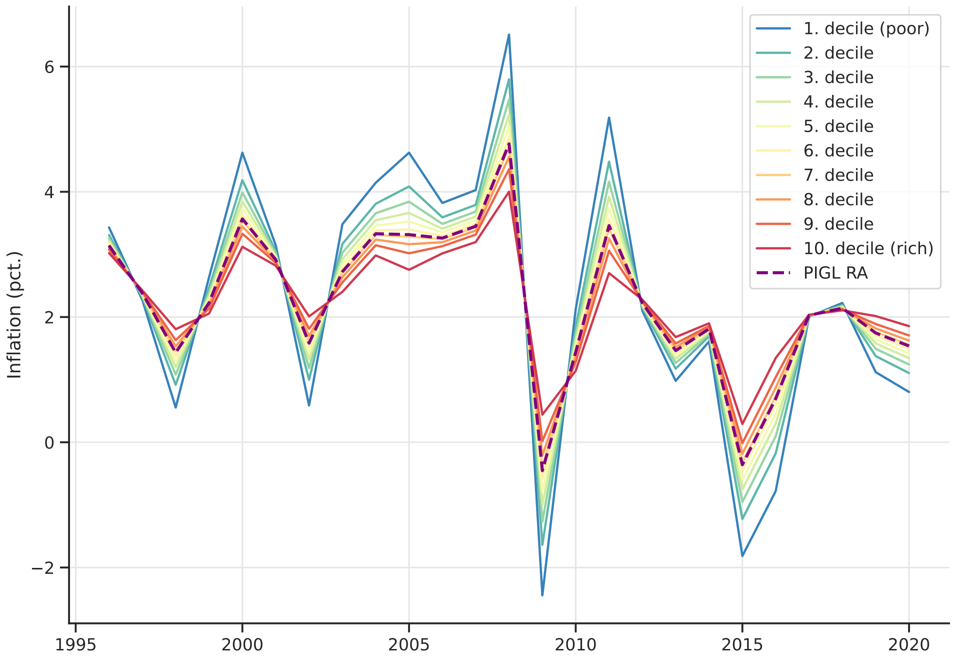

Figure 3. Generalized Sato-Vartia inflation under quasi separability by expenditure decile.

Notes. Inflation for each expenditure decile is calculated as the first difference in the price index of the PIGL representative agent over households within each respective decile. "PIGL RA" stands for the PIGL representative agent over *all* households.

living that we observe. We investigate these two channels in turn.

5.2.1 Different Goods Exposure

The geometric-mean form of the generalized superlative price indices in Corollary 1 makes a multiplicative decomposition of these indices straightforward: taking first differences of the log of the price index (16) immediately yields the contribution of each individual expenditure category to overall inflation. Figure 4 plots this decomposition for the poor and the rich and highlights the main contributing expenditure categories to inflation since the start of our sample.[11]

Panel (a) in Figure 4 shows that the two primary drivers of inflation for the poor are the expenditure categories "food at home" and "gasoline and utilities", which together dwarf all other consumption categories combined. The decomposition also reveals that the high inflation volatility of the poor stems from their large exposure to price changes in "gasoline and utilities", which fluctuates greatly from year to year. By contrast, panel (b) instead identifies housing as the main inflation driver for the rich, as seen by the contribution of "owned dwellings". The contribution of any individual category for rich households is much less pronounced, however. This is particularly the case for "food at home" and "gasoline and utilities", thus resulting in a more stable inflation rate for the rich.

[11] Figure B.6 in Appendix B shows the contribution from all expenditure categories to the inflation of the rich and poor.

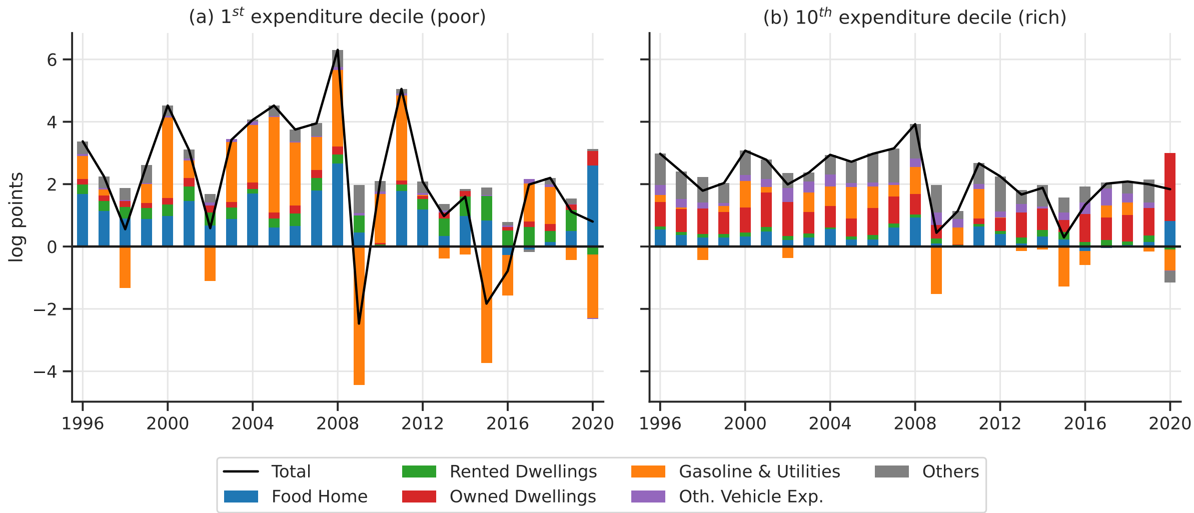

Figure 4. Inflation decomposition by expenditure categories.

5.2.2 Different Substitution Behavior

To investigate how substitution behavior affects changes in the cost of living, it is useful to consider a simple behavioral decomposition of the cost-of-living index. Specifically, if households do not substitute at all, consumed quantities of individual goods remain fixed over time. Households' cost-of-living indices should behave exactly like standard Laspeyres indices in that case, with the only differences across households coming from relative quantity differences in their base-period consumption baskets. The log difference between a generalized superlative index and a corresponding Laspeyres index therefore gives a measure of product substitution away from the base-period consumption basket as prices change. The Laspeyres price index is readily obtained as $P_{Lt} = \sum_j w_{js}(p_{jt}/p_{js})$ and, as shown in Appendix A.2 and by Balk (2004), this index can be written on a geometric-mean form as

$$P_{Lt} = \prod_{j \in J} \left(\frac{p_{jt}}{p_{js}} \right)^{\delta^L_{jt}}, \qquad \text{where} \qquad \delta^L_{jt} \equiv \frac{w_{js}\, L\left(\frac{p_{jt}}{p_{js}}, P_{Lt}\right)}{\sum_{i \in J} w_{is}\, L\left(\frac{p_{it}}{p_{js}}, P_{Lt}\right)}. \tag{20}$$

(As before, $L(\cdot,\cdot)$ denotes the logarithmic mean here.) Paired with the generalized superlative index (16), we then obtain the decomposition

$$\ln P_t = \underbrace{\sum_{j \in J} \delta^L_{jt} \ln \left(\frac{p_{jt}}{p_{js}} \right)}_{\text{Laspeyres price index}} + \underbrace{\sum_{j \in J} \left(\chi_{jt} - \delta^L_{jt} \right) \ln \left(\frac{p_{jt}}{p_{js}} \right)}_{\text{Product substitution}}. \tag{21}$$

Equation (21) separates the overall change in the cost of living into mechanical price changes on the base-period consumption basket in the first sum and the overall substitution effect in the second sum.

Figure 5 plots the decomposition (21) of the Generalized Sato-Vartia price index for the first

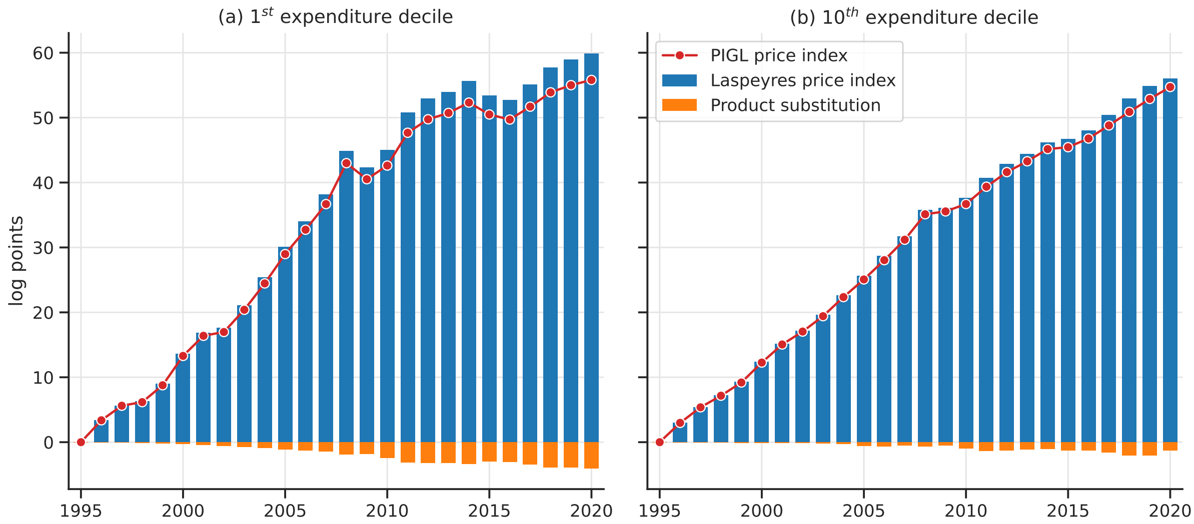

Figure 5. Decomposition of the Generalized Sato-Vartia price index into the Laspeyres price index and product substitution.

Notes. Decomposition according to Equation (21). The Generalized Sato-Vartia price index for each expenditure decile denotes the index of the representative agent within each decile.

and tenth expenditure deciles. The overwhelming majority of the change in the cost of living is caused by pure price changes on the base-period baskets. This is unsurprising given that Laspeyres indices are first-order approximations of true cost-of-living indices (see for instance Deaton and Muellbauer, 1980b, ch. 7.1). Despite the small role for product substitution in shaping the overall price indices, Figure 5 nevertheless highlights a more prominent substitution margin among the poor than among the rich. This contrasts with the findings of for instance Argente and Lee (2021), who argue that low-income households experience higher inflation than high-income households, partly because the latter group has more options to substitute away from high-quality goods (luxury goods, presumably).[12]

To what extent do these differences affect the disparities in inflation and the cost of living? Figure 6 plots the log difference between the decompositions for the first and tenth expenditure deciles. Over the last 20 years, the increase in the cost of living for the poor is approximately between 1 and 8 percent larger than that of the rich. Behavioral differences limit this gap somewhat: more substitution among the poor reduces the difference by around 2 percentage points throughout the last decade. This effect is seemingly stable, however, and most swings from year to year instead come from pure price changes in the different goods households consume.

[12] A key difference to Argente and Lee (2021) is that we evaluate substitution in *quantities consumed* whereas Argente and Lee define product substitution to be *changes in expenditure shares*. Another distinction is that Argente and Lee use scanner data from the retail sector and therefore primarily consider food and other groceries, whereas our CEX data includes virtually all household consumption.

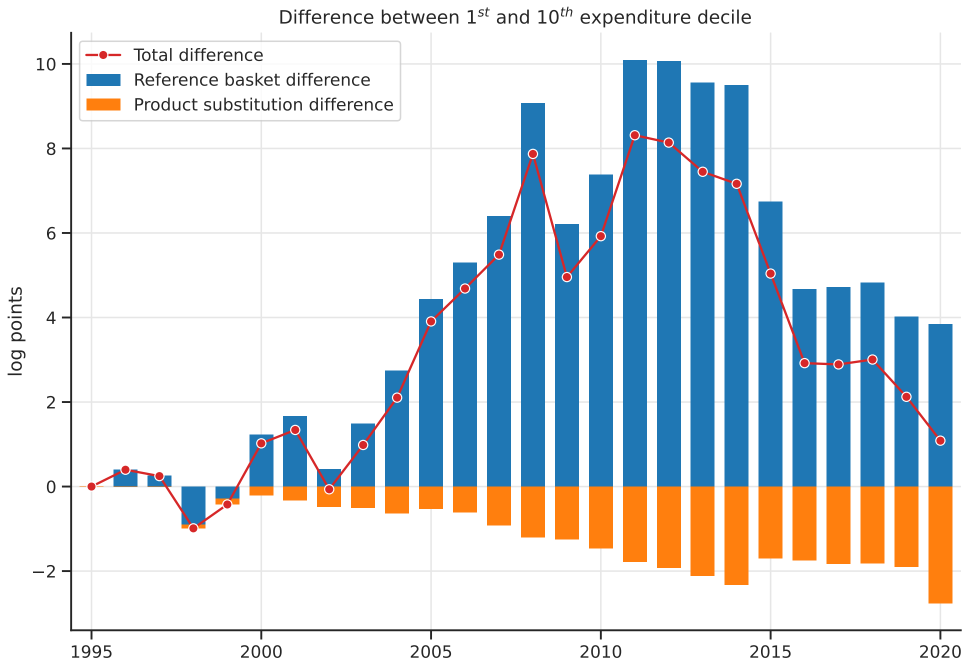

Figure 6. Difference of the decomposed Generalized Sato-Vartia price index between the first and tenth expenditure decile.

5.2.3 Taking Stock

The two decompositions above suggest that the higher inflation volatility for consumption-poor households is due to differences in types of goods that people consume across the consumption distribution. Poorer households allocate a larger share to more price-volatile necessity goods, especially "food at home" and "gasoline and utilities". Behavioral differences help to dampen inflation level differences, as poorer households substitute more than the rich, but does seem to affect the differences in inflation volatility.

6 Why We Bother: Comparison With Previous Methods

A natural question arises at this point. If all we do in our main results is to construct and decompose cost-of-living indices across the consumption distribution, then why do we bother with developing a new methodology for it? Previous papers studying inflation inequality simply split households into different groups and compute homothetic indices for each group separately based on observed expenditure shares within these groups (examples include Broda and Romalis, 2009, Jaravel, 2019, Argente and Lee, 2021, and Klick and Stockburger, 2021). These group-specific indices do not require any estimation or classification of goods, and nothing stops us from conducting all the analyses in Section 5 with this method instead. So why do we jump through all the hoops needed to get our approach to work?

One reason is that our methodology avoids a particular bias which is present in the group-specific index approach. Recall that a cost-of-living index is defined as the percentage change in consumption expenditures needed to maintain a *constant* standard of living when prices change. Consequently, a true cost-of-living index only reflects price changes and subsequent substitution

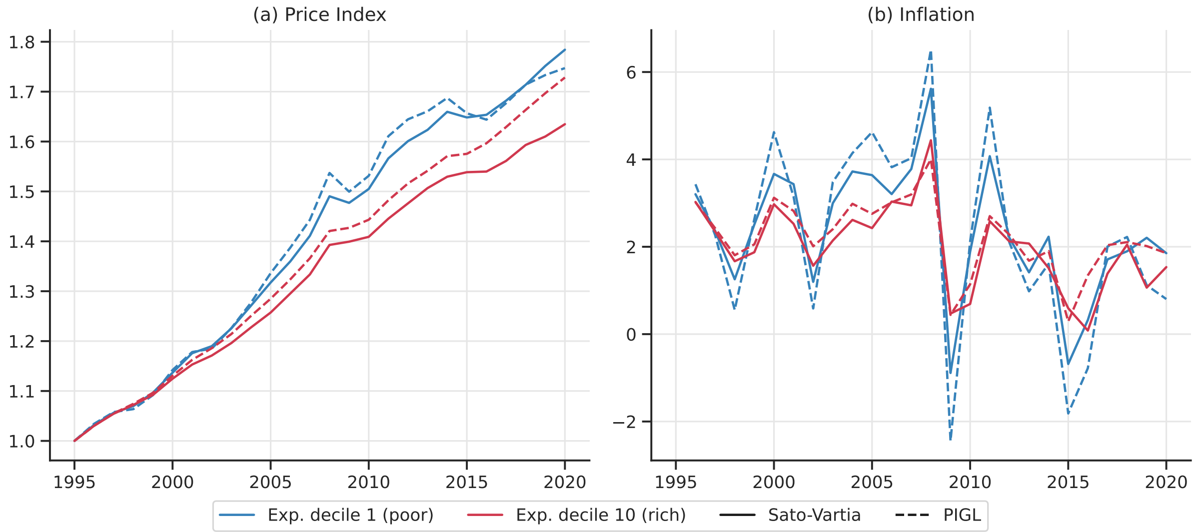

Figure 7. Comparison of group-specific Sato-Vartia cost-of-living index with PIGL cost-of-living index.

Notes. The group-specific Sato-Vartia index is computed based on a grouping of households into expenditure deciles of the respective year. The PIGL price index for each expenditure decile is calculated as the price index of the PIGL representative agent over households within each respective decile.

effects, while ignoring income effects on consumer behavior. The corresponding index weights are therefore based not on actual observed expenditure shares, but on hypothetical ***compensated*** expenditure shares (that is, the shares that prevail at observed prices if utility remains fixed at its base-period level). This distinction does not matter under homothetic preferences, because compensated shares equal observed shares in this case. If preferences are nonhomothetic, however, then observed expenditure shares also capture income effects related to changes in the standard of living over time. This drives a wedge between observed and compensated shares, thereby causing a bias in price indices that rely on the former.[13] Our approach overcomes this issue by relying on compensated rather than observed expenditure shares.

We examine the magnitude of this bias by comparing the Generalized Sato-Vartia index to the group-specific price index approach. To that end, we partition the households in the CEX data into year-specific consumption deciles and use the observed average expenditure shares w_{jt} within these groups to compute decile-specific Sato-Vartia indices $\ln P_t = \sum_j \delta_{jt} \ln(p_{jt}/p_{js})$, where $\delta_{jt} = L(w_{jt}, w_{js})/\sum_i L(w_{it}, w_{is})$. Figure 7 plots the price indices and the inflation rates for the top and bottom consumption deciles using this approach against those of the Generalized Sato-Vartia index. Table 3 summarizes average inflation rates and inflation volatility over the full sample period across the consumption distribution for both approaches.

Compared to the Generalized Sato-Vartia index, the group-specific index approach consistently over-estimates inflation at the lower end of the distribution and under-estimates it at the upper end. The size of the bias ranges from a 0.07 percentage point higher average annual inflation rate for the lowest decile to a 0.23 percentage point lower inflation rate for the top decile. These differences generate a larger inflation gap across the distribution: the average annual inflation

[13] Oulton (2008) labels this a “path-dependence bias” and provides a more detailed discussion on the issue.

Table 3. Inflation rate levels and dispersion over time across the consumption distribution for two different approaches, 1996–2020.

	Generalized Sato-Vartia index (PIGL)				Group-specific homothetic Sato-Vartia indices					
	Level		Dispersion		Level			Dispersion		
Decile	Mean (%)	Relative to top decile (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)	Mean (%)	Relative to top decile (pp. diff.)	Relative to PIGL (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)	Relative to PIGL (x Std Dev)
1	2.28	0.06	2.14	2.51	2.35	0.36	0.07	1.48	1.51	0.69
2	2.26	0.04	1.76	2.06	2.33	0.34	0.07	1.41	1.45	0.80
3	2.25	0.03	1.59	1.86	2.31	0.32	0.06	1.51	1.54	0.95
4	2.24	0.03	1.46	1.71	2.30	0.31	0.05	1.47	1.51	1.01
5	2.24	0.02	1.36	1.60	2.24	0.25	0.00	1.36	1.39	1.00
6	2.24	0.02	1.28	1.50	2.24	0.25	0.01	1.32	1.35	1.03
7	2.23	0.02	1.20	1.40	2.22	0.23	−0.02	1.29	1.32	1.08
8	2.23	0.01	1.12	1.31	2.19	0.20	−0.03	1.20	1.23	1.07
9	2.22	0.01	1.02	1.20	2.15	0.16	−0.08	1.12	1.14	1.10
10	2.22	0.00	0.85	1.00	1.99	0.00	−0.23	0.98	1.00	1.15

Notes. Arithmetic mean and standard deviation of annual inflation over the time period 1996–2020. Under inflation rate levels, the "Relative to top decile" columns show the percentage point difference in the average annual inflation rate to that of the tenth consumption decile while the "Relative to PIGL" column shows the difference in average inflation between the group-specific Sato-Vartia index and the PIGL index in the same expenditure decile. Under inflation rate dispersion, the same columns show the standard deviation of annual inflation as a multiple of that of the tenth consumption decile and of the same decile for the PIGL index.

rate in the bottom decile is 2.35 while the corresponding number for the top decile is 1.99 percent, a 0.36 percentage point difference. The group-specific Sato-Vartia indices therefore produce a divergence in the cost of living between the poorest and the richest households, as shown in panel (a) of Figure 7.

The group-specific approach also generates a more stable inflation rate over both time and consumption groups, as shown in panel (b) of Figure 7. Compared to the Generalized Sato-Vartia index, the standard deviation of inflation over time is now 31 percent lower for the bottom decile and 15 percent higher for the top decile. This evidently reduces the volatility gap: the relative inflation rate volatility between the poorest and richest households is now 1.51, considerably lower than the 2.51 found with the Generalized Sato-Vartia index.

Taken together, these results point to the importance of purging price indices from any income effects on household behavior. Failing to do so in our application increases the inflation rate difference between the top and bottom of the consumption distribution by 0.3 percentage points and generates biased disparities in the cost of living. Interestingly, 0.3 percentage points is close to the range of inflation rates found in papers employing the group-specific approach. Perhaps most comparable are Jaravel (2019) and Klick and Stockburger (2021), who construct income-specific Törnqvist indices using similar CEX-CPI data. These authors find a 0.35–0.38 percentage point inflation rate difference between the top and bottom 20–25 percent of the income distribution. Our Generalized Sato-Vartia index exhibits similar inflation differences for the years considered by these authors, but the size of the bias found here nevertheless prompts the question of how much of their findings are driven by the same bias.

7 The (Un)Importance of the Separability Assumption

The results above rely on an implementation method that rests on one key assumption: that goods can be split up into two separate consumption bundles, one that includes all necessity goods and one that includes all luxury goods. How strong is this assumption in our application? To answer this question, we finish the paper with a robustness check in which we compare the baseline results in Section 5 with those of a demand system estimation on a fully parametrized model.

To that end, we parametrize the cost functions $B(\boldsymbol{p})$ and $D(\boldsymbol{p})$ and perform a nonlinear GMM estimation of the system of the expenditure shares defined by Equation (8). Since the Sato-Vartia index is the Konüs index corresponding to the CES expenditure function, we parametrize the cost functions of the two baskets as such:

$$B(\boldsymbol{p}) = \left(\sum_{j \in J} \omega_j p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \qquad \text{and} \qquad D(\boldsymbol{p}) = \left(\sum_{j \in J} \theta_j p_j^{1-\varphi} \right)^{\frac{1}{1-\varphi}}, \tag{22}$$

where $\sigma, \varphi > 0$ and where the share parameters satisfy $\sum_{j \in J} \omega_j = \sum_{j \in J} \theta_j = 1$ and $\omega_j, \theta_j \geq 0$ for all $j \in J$. (A similar parametrization is also considered by Alder, Boppart and Müller, 2022.) The possibility of binding boundary constraints on the parameters generates a nonstandard asymptotic distribution, so neither the bootstrap nor the standard covariance matrix produces consistent standard errors in this case (see Andrews, 1999, 2000). Although this complicates

Table 4. GMM estimates of the preference parameters.

	Weak separability (Sato-Vartia)	Full demand system (CES)
ε	0.677	0.685
	(0.004)	(0.004)
γ	0.211	0.505
	(0.023)	(0.018)
$\widetilde{\nu}$	327.271	346.736
	(13.358)	(11.793)
σ		0.050
		(0.012)
φ		0.360
		(0.006)
Observations	74,372	74,372
RMSE	0.1487	0.1494

Notes. Robust standard errors in parentheses. "RMSE" refers to the root-mean-square error of the expenditure share on the D good: $\sqrt{\sum_h (w_{Dh} - \widehat{w}_{Dh})^2 / N}$. Observation are weighted by their CEX sampling weights.

statistical inference, the point estimates of the GMM estimator still remains consistent when a true parameter lies on its boundary (Andrews, 2002). Our GMM estimation can therefore be seen as something akin to a calibration rather than a full estimation. As an initial parameter guess, we set ε, γ and $\widetilde{\nu}$ to the values in Table 1 and distribute ω_j (θ_j) equally among the goods that are classified as luxuries (necessities) in the quasi separability case while the remaining share parameters are set to zero.

Table 4 and Figure 8 show the point estimates associated with the local minimum obtained from our initial guess.[14] The key parameter with respect to nonhomotheticity, ε, and the scale parameter $\widetilde{\nu}$ are virtually unchanged. The CES weights ω_j and θ_j are also fully in line with the quasi separability assumption: all necessities from our baseline approach hold zero weight in the luxury basket B, and vice versa for luxuries. The main difference relates to γ. The higher value of γ in the full demand system lowers the elasticity of substitution between B and D for everyone, but more so for the poor than for the rich. For instance, the elasticity of substitution for a household with a 90 percent expenditure share on the D basket is 4.98 in the baseline and 2.12 in the full demand system. The corresponding numbers for a household with a 10 percent expenditure share on D are 0.84 and 0.52. Relative to our baseline approach, the higher value of γ therefore reduces the importance of substitution behavior in explaining any cost-of-living differences between rich and poor households.

[14] The usual caveats of nonlinear estimation apply, in particular that the GMM objective function may exhibit multiple local minima, with no guarantee that our solution is the global minimum. In principle, we could get around this issue by a grid search. There are 45 parameters to estimate, however, and the associated curse of dimensionality makes such a grid search infeasible in practice.

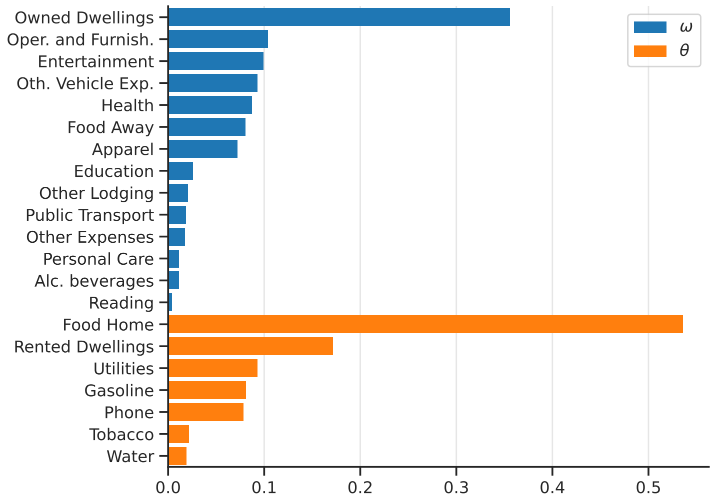

Figure 8. Point estimates for the CES share parameters ω_j and θ_j.

Notes. Results are for the closest local minimum to the quasi-separable case which has been used as initial guess for the parameters. Point estimates on the zero boundary are omitted.

Overall, the parameter estimates from the estimated demand system are fully in line with the quasi separability assumption in the baseline approach. We plot a comparison between the cost-of-living indices resulting from the two approaches in Figure 9 to make sure that the higher value of γ does not radically alter our baseline findings.[15] Fortunately, the two cases are barely distinguishable. Thus, taken at face value, the findings in this section seem to fully justify the quasi separability assumption for the dataset that we use.

8 Conclusion

We derive a nonhomothetic cost-of-living index which allows us to describe inflation heterogeneity along the full expenditure distribution. The cost-of-living index is microfounded with PIGL preferences and we show that it can be computed without a full demand system estimation if quasi separability of consumption goods into necessary and luxury goods is imposed. The theoretical rigor and practical simplicity of our index makes it especially appealing compared to other approaches previously taken in the literature on inflation heterogeneity.

The price index generalizes several classes of homothetic price indices, some of them particularly interesting for their superlative property. We present results for generalized Sato-Vartia, Törnqvist, geometric Walsh, Theil, Fisher and arithmetic Walsh price indices. Homothetic indices such as the Sato-Vartia and Törnqvist are used in previous empirical studies to approximate inflation heterogeneity by computing separate price indices for different income groups. We show that this approach can be rationalized through the lens of our framework but the usual caveats of the group-specific approach still apply.

Our empirical results show that from 1996 to 2020 there was a substantial heterogeneity in inflation between poorer and richer households in the US. The particular striking result we find is

[15] A similar comparison for the inflation rate is shown in Figure B.9 in Appendix B.

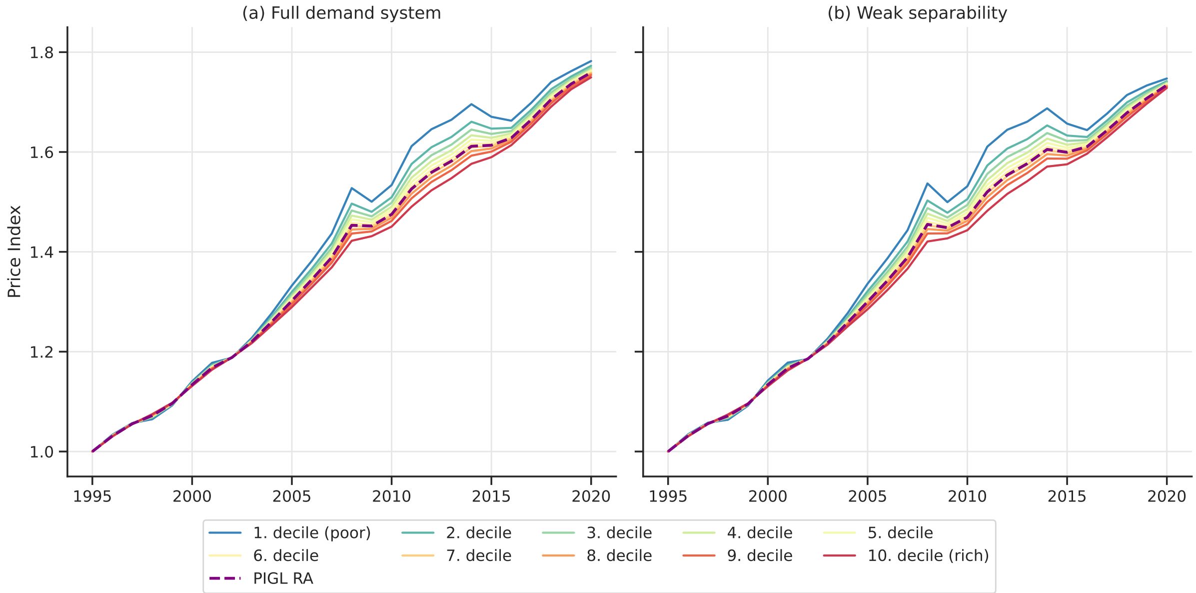

Figure 9. Comparison of the Generalized Sato-Vartia price index for the full demand system and under quasi separability.

that while mean inflation is around 2.25 percent for everyone, the standard deviation of inflation has been 2.14 for the poor compared to 0.85 for the rich. Thus, inflation volatility is 2.5 times higher for the poor.

We find that poorer households are much more exposed to the highly volatile inflation rates of food, gasoline and utilities compared to the rich. We furthermore show that substitution behavior is only of second-order importance. Our findings hence suggest that in order to uncover the fundamental drivers of inflation inequality it is first and foremost important to understand *why* households make the consumption choices they do and what the explanation for inflation heterogeneity across product groups is.

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Appendix A Proofs and Extensions

A.1 Proof of Proposition 1

Proof. Inverting the indirect utility function (4) gives the expenditure function

$$c(u, \boldsymbol{p}) = \left[1 + \varepsilon\left(u + \frac{\nu}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}).$$

Suppose that the reference utility u corresponds to the expenditure level in some base period s such that $c(u, \boldsymbol{p}_s) = e_s$ and $u \equiv V(e_s, \boldsymbol{p}_s)$. Using the indirect utility function (4) to substitute this into some period-t expenditure function and rearranging terms yields

$$\begin{aligned}
c(u, \boldsymbol{p}_t) &= e_s\left[1 + \frac{\varepsilon\,\nu}{\gamma}\left(\frac{B(\boldsymbol{p}_s)}{e_s}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p}_s)}{B(\boldsymbol{p}_s)}\right)^{\gamma}\left\{\left(\frac{D(\boldsymbol{p}_t)}{D(\boldsymbol{p}_s)}\right)^{\gamma}\left(\frac{B(\boldsymbol{p}_t)}{B(\boldsymbol{p}_s)}\right)^{-\gamma} - 1\right\}\right]^{\frac{1}{\varepsilon}}\frac{B(\boldsymbol{p}_t)}{B(\boldsymbol{p}_s)} \\
&= e_s\left[1 + \frac{\varepsilon\, w_{Ds}}{\gamma}\left\{\left(\frac{P_{Dt}}{P_{Bt}}\right)^{\gamma} - 1\right\}\right]^{\frac{1}{\varepsilon}} P_{Bt} \\
&= e_s\left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma}\right)P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}\cdot\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}},
\end{aligned}$$

where the second equality uses $P_{Bt} \equiv B(\boldsymbol{p}_t)/B(\boldsymbol{p}_s)$, $P_{Dt} \equiv D(\boldsymbol{p}_t)/D(\boldsymbol{p}_s)$ and the expenditure share (5). By the Konüs definition (1), the cost-of-living index is then

$$P_t^{PIGL} = \left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma}\right)P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}\cdot\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}} = \widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}. \tag{A.1}$$

Since $\widetilde{P}_t$ has a CES form, we may define hypothetical budget shares corresponding to this price function by

$$\psi_{Bt} \equiv \left(1 - \frac{\varepsilon w_{Ds}}{\gamma}\right)\left(\frac{P_{Bt}}{\widetilde{P}_t}\right)^{\gamma} \qquad \text{and} \qquad \psi_{Dt} \equiv \frac{\varepsilon w_{Ds}}{\gamma}\left(\frac{P_{Dt}}{\widetilde{P}_t}\right)^{\gamma}, \tag{A.2}$$

with $\psi_{Bs} = 1 - \varepsilon w_{Ds}/\gamma$ and $\psi_{Ds} = \varepsilon w_{Ds}/\gamma$. These shares ensure that the price index remains at the same utility level; $w_{Ds}(P_{Dt} \,/\, \widetilde{P}_t)^{\gamma}$ is the expenditure share of the D basket at period-t prices that prevails at the same utility level as w_{Ds}. To see this, use the expenditure share (5) to get

$$\begin{aligned}
w_{Dt} &= \nu\left(\frac{B(\boldsymbol{p}_t)}{e_t}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p}_t)}{B(\boldsymbol{p}_t)}\right)^{\gamma} && \text{(by (5))} \\
&= \nu\left(\frac{B(\boldsymbol{p}_s)}{e_s}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p}_s)}{B(\boldsymbol{p}_s)}\right)^{\gamma}\left(\frac{e_s P_{Dt}^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}}{e_t}\right)^{\varepsilon} && \text{(by (3))}
\end{aligned}$$

$$= w_{Ds}\left(\frac{P_{Dt}^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}}{P_t Q_t}\right)^{\varepsilon} \qquad \text{(by (5) and } e_t/e_s = P_t Q_t\text{)}$$

$$= w_{Ds}\left(\frac{P_{Dt}}{\widetilde{P}_t}\right)^{\gamma} Q_t^{-\varepsilon} \qquad \text{(by (A.1)).}$$

The third equality uses the decomposition $e_t/e_s = P_t Q_t$, where P_t is the Konüs price index and Q_t the corresponding quantity index. Along the same indifference curve as w_{Ds}, we necessarily have $Q_t = 1$ for all t, and the result then immediately follows. Equation (A.2) allows us to write $\widetilde{P}_t$ as a Sato-Vartia index. The procedure is the same as in the standard case: solve for $\widetilde{P}_t$ from the shares in Equation (A.2), take logs, multiply by the difference in shares over time, sum over both B and D and solve for $\widetilde{P}_t$. For $C \quad \{B, D\}$, the first two steps yields

$$\ln \widetilde{P}_t = \ln P_{Ct} - \frac{1}{\gamma}\ln\left(\frac{\psi_{Ct}}{\psi_{Cs}}\right) \qquad\qquad -\frac{1}{\gamma} = \frac{\ln \widetilde{P}_t - \ln P_{Ct}}{\ln \psi_{Ct} - \ln \psi_{Cs}}.$$

Multiplying both sides by $\psi_{Ct} - \psi_{Cs}$, summing over both $C \quad \{B, D\}$, and rearranging terms results in

$$\ln \widetilde{P}_t \sum_{C \quad \{B,D\}} \frac{\psi_{Ct} - \psi_{Cs}}{\ln \psi_{Ct} - \ln \psi_{Cs}} = \sum_{C \quad \{B,D\}} \frac{\psi_{Ct} - \psi_{Cs}}{\ln \psi_{Ct} - \ln \psi_{Cs}} \ln P_{Ct}.$$

Then solving for $\widetilde{P}_t$ yields

$$\widetilde{P}_t = P_{Dt}^{\phi_t} P_{Bt}^{1-\phi_t}, \qquad \text{where} \qquad \phi_t \equiv \frac{L(\psi_{Dt}, \psi_{Ds})}{L(\psi_{Dt}, \psi_{Ds}) + L(\psi_{Bt}, \psi_{Bs})}. \tag{A.3}$$

Plugging Equation (A.3) into Equation (A.1) gives the household-level price indices.

Because a representative level of expenditures e^{RA} exists over any group of households, group-level behavior is characterized by the same indirect utility function and expenditure function as household-level behavior.[16] Aggregate-level cost-of-living indices are therefore derived identically to above, with the only difference that group-level expenditure shares $\overline{w}_{Ds}$ and representative levels of expenditure e^{RA} are used instead of household-level ones. □

A.2 Proof of Corollary 1

Proof. The result is immediate by setting both P_{Dt} and P_{Dt} to either the ***quadratic-mean-of-order-r*** index (14) or the ***Theil-Sato*** index (15) and substituting these into the general PIGL index (11). We only need to rewrite (14) into a geometric-mean form. Balk (2004) does this for the Fisher ideal index ($r = 2$), and the generalization to any $r > 0$ is analogous. As in Corollary 1, define

$$P_{Lt} = \left[\sum_{j \quad J} w_{js}\left(\frac{p_{jt}}{p_{js}}\right)^{\frac{r}{2}}\right]^{\frac{2}{r}} \qquad \text{and} \qquad P_{Pt} = \left[\sum_{j \quad J} w_{jt}\left(\frac{p_{jt}}{p_{js}}\right)^{-\frac{r}{2}}\right]^{-\frac{2}{r}},$$

[16] This can be shown by substituting for $e^{RA} = \bar{e}\kappa^{-\frac{1}{\varepsilon}}$ in the aggregate expenditure share (9) and integrating to obtain the group-level indirect utility function.

such that the ***quadratic-mean-of-order-r*** index (14) can be written $P_t = \sqrt{P_{Lt}P_{Pt}}$. P_{Lt} weighs price changes by base-period expenditure shares while P_{Pt} uses current-period expenditure shares, and the definition nests the Laspeyres and Paasche indices as the special case where $r = 2$, thus motivating the L and P notation. By the definition of P_{Lt} and the logarithmic mean, it holds that

$$\begin{aligned} 0 &= \sum_{j \in J} w_{js} \left(\frac{p_{jt}}{p_{js}}\right)^{\frac{r}{2}} - P_{Lt}^{\frac{r}{2}} = \sum_{j \in J} w_{js} \left[\left(\frac{p_{jt}}{p_{js}}\right)^{\frac{r}{2}} - P_{Lt}^{\frac{r}{2}} \right] \\ &= \sum_{j \in J} w_{js}\, L\left((\tfrac{p_{jt}}{p_{js}})^{\frac{r}{2}},\, P_{Lt}^{\frac{r}{2}} \right) \ln \left(\frac{p_{jt}/p_{js}}{P_{Lt}}\right)^{\frac{r}{2}} \\ &= \frac{r}{2} \sum_{j \in J} \widetilde{w}_{Ljt} \left[\ln\left(\frac{p_{jt}}{p_{js}}\right) - \ln P_{Lt} \right], \end{aligned}$$

with $\widetilde{w}_{Ljt}$ defined as in Corollary 1. Solving for $\ln P_{Lt}$, we get

$$\ln P_{Lt} = \sum_{j \in J} \frac{\widetilde{w}_{Ljt}}{\sum_{i \in J} \widetilde{w}_{Lit}} \ln \left(\frac{p_{jt}}{p_{js}}\right).$$

Identical steps for P_{Pt} yields

$$\ln P_{Pt} = \sum_{j \in J} \frac{\widetilde{w}_{Pjt}}{\sum_{i \in J} \widetilde{w}_{Pit}} \ln \left(\frac{p_{jt}}{p_{js}}\right),$$

with $\widetilde{w}_{Pjt}$ defined as in Corollary 1. Substituting these into the overall index P_t yields

$$P_t = \prod_{j \in J} \left(\frac{p_{jt}}{p_{js}}\right)^{\delta_{jt}}, \qquad \text{where} \qquad \delta_{jt} \equiv \frac{1}{2}\left[\frac{\widetilde{w}_{Ljt}}{\sum_i \widetilde{w}_{Lit}} + \frac{\widetilde{w}_{Pjt}}{\sum_i \widetilde{w}_{Pit}} \right],$$

and we are done. □

A.3 Proof of Proposition 3

Proof. If preferences are of the Cobb-Douglas form $V(e, \boldsymbol{p}) = \left[\frac{e}{B(\boldsymbol{p})^{1-\nu} D(\boldsymbol{p})^{\nu}}\right]$, then the cost-of-living index is $P_t = P_{Bt}^{1-\nu} P_{Dt}^{\nu}$ by Proposition 1. If P_{Bt} and P_{Dt} are Törnqvist indices, Corollary 1 and Assumption 1 allow us to write this index as

$$P_t = \prod_{j \in J_B} \left(\frac{p_{jt}}{p_{js}}\right)^{(1-\nu)\delta_{jt}^B} \prod_{j \in J_D} \left(\frac{p_{jt}}{p_{js}}\right)^{\nu \delta_{jt}^D}, \qquad \delta_{jt}^C = \frac{w_{js}^C + w_{jt}^C}{2}, \quad C \in \{B, D\}, \tag{A.4}$$

where J_B and J_D denote the sets of goods in B and D, respectively. (Under the quasi separability assumption, it holds that $J_B \cup J_D = J$ and $J_B \cap J_D = \varnothing$.) Meanwhile, the standard Törnqvist index reads

$$P_t = \prod_{j \quad J} \left(\frac{p_{jt}}{p_{js}} \right)^{\delta_{jt}}, \qquad \delta_{jt} = \frac{w_{js} + w_{jt}}{2}. \tag{A.5}$$

Equations (A.4) and (A.5) are equal if $(1 - \nu)\delta_{jt}^B = \delta_{jt}$ for all $j \quad J_B$ and $\nu\delta_{jt}^D = \delta_{jt}$ for all $j \quad J_D$. Under Cobb-Douglas preferences, ν is the homothetic and time-invariant expenditure share on D, w_D. Under the quasi separability assumption, the total expenditure share on good $j \quad J_C$, $C \quad \{B, D\}$, is given by $w_j = w_C \, w_j^C$. Thus,

$$\begin{aligned} (1 - \nu)\delta_{jt}^B &= w_B \frac{w_{js}^B + w_{jt}^B}{2} = \frac{w_{js} + w_{jt}}{2} = \delta_{jt}, \qquad j \quad J_B, \\ \nu\delta_{jt}^D &= w_D \frac{w_{js}^D + w_{jt}^D}{2} = \frac{w_{js} + w_{jt}}{2} = \delta_{jt}, \qquad j \quad J_D, \end{aligned}$$

and it follows that the Törnqvist index under quasi separability (A.4) is the same as the standard Törnqvist index (A.5). □

A.4 Allowing for Heterogeneity in Tastes

Redding and Weinstein (2020) stress the importance of accounting for heterogeneity in tastes for the cost of living and it is possible to extend the baseline framework to allow for this. Following Cravino, Levchenko and Rojas (2022), let the preferences of household h be characterized by an indirect utility function of the form

$$V_h(e_h, \boldsymbol{p}) = \frac{1}{\varepsilon}\left[\left(\frac{e_h}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] - \frac{\nu_h}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right], \tag{A.6}$$

where the only difference to the PIGL specification in Equation (4) is that we allow for a time-invariant taste parameter ν_h that varies across households. As before, the expenditure share of the latent good with price function $D(\cdot)$ is given by Roy's identity as

$$w_{Dh} = \nu_h \left(\frac{B(\boldsymbol{p})}{e_h}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma},$$

and the corresponding aggregate expenditure share over a measure N of households is now

$$\overline{w}_D = \bar{\nu} \left(\frac{B(\boldsymbol{p})}{\bar{e}}\right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} \kappa, \qquad \text{where} \qquad \kappa \equiv \frac{1}{N}\int_0^N \frac{\nu_h}{\bar{\nu}} \left(\frac{e_h}{\bar{e}}\right)^{1-\varepsilon} dh,$$

and where $\bar{\nu} = \frac{1}{N}\int_0^N \nu_h dh$ denotes the average ν_h. A representative agent with expenditure level $e^{RA} = \bar{e}\kappa^{-\frac{1}{\varepsilon}}$ therefore exists and incorporates any deviations from the mean taste level. Substituting back into the aggregate expenditure share $\overline{w}_D$ and integrating back yields aggregate-level behavior characterized by the indirect utility function

$$V(e^{RA}, \boldsymbol{p}) = \frac{1}{\varepsilon}\left[\left(\frac{e^{RA}}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] - \frac{\bar{\nu}}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right],$$

with the corresponding expenditure function

$$c(u^{RA}, \boldsymbol{p}) = \left[1 + \varepsilon\left(u^{RA} + \frac{\overline{\nu}}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}).$$

This expenditure function is independent of individual taste parameters ν_h. We can therefore follow the same steps as in Appendix A.1 to derive an identical price index as in Proposition 1. Again, this index is only a function of the base-period expenditure share for the D basket, price indices P_{Dt} and P_{Bt}, and the parameters ε and γ. Heterogeneity in the taste parameters ν_h only affect the price index indirectly to the extent that they affect expenditure shares. Whenever expenditure shares are observed in the data, we therefore do not need to know these individual tastes to compute the price index. Since this holds for any measure N of households, it also for groups consisting of only one person, so the price index applies at the household level.

Taste heterogeneity also poses no challenge with respect to estimating ε and γ. Since PIGL preferences aggregate consistently, it is possible to estimate these parameters from a aggregate data without any aggregation bias. Therefore, taking an aggregate time series and estimating

$$\overline{w}_{Dt} = \overline{\nu}\left(\frac{P_{Bt}}{\bar{e}_t}\right)^{\varepsilon}\left(\frac{P_{Dt}}{P_{Bt}}\right)^{\gamma}\kappa,$$

where $\overline{\nu}\kappa$ is just a standard regression constant, is sufficient and this avoids the need to estimate all the household-level effects ν_h.

A.5 Allowing for Hump-Shaped Expenditure Shares

Banks, Blundell and Lewbel (1997) stress the importance of allowing for hump-shaped expenditure shares to match the microeconomic data and it is possible to extend the baseline framework to allow for this at the household level. Following Alder, Boppart and Müller (2022), let preferences be characterized by an indirect utility function of the form

$$V(e, \boldsymbol{p}) = \frac{1}{\varepsilon}\left[\left(\frac{e - A(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right], \tag{A.7}$$

where the only difference to the PIGL specification in Equation (4) is the addition of a linearly homogeneous function $A(\boldsymbol{p})$ of prices. The expenditure shares of the three latent goods with price functions $A(\cdot)$, $B(\cdot)$ and $D(\cdot)$ are given by Roy's identity as

$$w_A = \frac{A(\boldsymbol{p})}{e}, \tag{A.8}$$

$$w_B = \left(1 - \frac{A(\boldsymbol{p})}{e}\right)\left[1 - \nu\left(\frac{B(\boldsymbol{p})}{e - A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\right], \tag{A.9}$$

$$w_D = \left(1 - \frac{A(\boldsymbol{p})}{e}\right)\nu\left(\frac{B(\boldsymbol{p})}{e - A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}. \tag{A.10}$$

The shares w_j^A, w_j^B and w_j^D of total A, B and D expenditures allocated to an individual good j are given as before by $w_j^C = p_j C_j(\boldsymbol{p})/C(\boldsymbol{p})$, $C \in \{A, B, D\}$. Together with Equations (A.8) to (A.10), this implies an expenditure share w_j of good j in total expenditures of the form

$$w_j = p_j \left\{ \frac{A(\boldsymbol{p})}{e} \frac{A_j(\boldsymbol{p})}{A(\boldsymbol{p})} + \left(1 - \frac{A(\boldsymbol{p})}{e}\right) \left[\frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} + \left(\frac{D_j(\boldsymbol{p})}{D(\boldsymbol{p})} - \frac{B_j(\boldsymbol{p})}{B(\boldsymbol{p})} \right) \nu \left(\frac{B(\boldsymbol{p})}{e - A(\boldsymbol{p})} \right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})} \right)^{\gamma} \right] \right\}. \tag{A.11}$$

Since the first term on the right-hand side of (A.11) is decreasing in e while the second term can be either increasing or decreasing in e, this allows for expenditure shares that are non-monotonic in expenditures. The derivation of the exact price index of (A.7) is virtually identical to the PIGL case in Appendix A.1. The corresponding expenditure function of (A.7) is

$$c(u, \boldsymbol{p}) = \left[1 + \varepsilon \left(u + \frac{\nu}{\gamma} \left\{ \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})} \right)^{\gamma} - 1 \right\} \right) \right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}) + A(\boldsymbol{p}).$$

Suppose again that the reference utility u is that corresponding to the expenditure level in some base period s such that $c(u, \boldsymbol{p}_s) = e_s$ and $u \equiv V(e_s, \boldsymbol{p}_s)$. Using the indirect utility function (A.7) to substitute this into some period-t expenditure function, rearranging terms, and using $P_{Ct} = C(\boldsymbol{p}_t)/C(\boldsymbol{p}_s)$ together with Equations (A.8) to (A.10) yields

$$c(u, \boldsymbol{p}_t) = e_s \left\{ (1 - w_{As}) \left[\left(1 - \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} \right) P_{Bt}^{\gamma} + \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} P_{Dt}^{\gamma} \right]^{\frac{1}{\gamma} \cdot \frac{\gamma}{\varepsilon}} P_{Bt}^{1 - \frac{\gamma}{\varepsilon}} + w_{As} P_{At} \right\},$$

and it follows that the price index is

$$P_t = (1 - w_{As}) \widetilde{P}_t^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1 - \frac{\gamma}{\varepsilon}} + w_{As} P_{At}$$

where

$$\widetilde{P}_t \equiv \left[\left(1 - \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} \right) P_{Bt}^{\gamma} + \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} P_{Dt}^{\gamma} \right]^{\frac{1}{\gamma}}.$$

Writing $\widetilde{P}_t$ as a Sato-Vartia index, we finally obtain the household-level cost-of-living index

$$P_t^{IA} = (1 - w_{As}) P_{Dt}^{\frac{\gamma \phi_t}{\varepsilon}} P_{Bt}^{1 - \frac{\gamma \phi_t}{\varepsilon}} + w_{As} P_{At},$$

where ϕ_t is a Sato-Vartia weight as in Proposition 1 with

$$\psi_{Bt} \equiv \left(1 - \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} \right) \left(\frac{P_{Bt}}{\widetilde{P}_t} \right)^{\gamma} \qquad \text{and} \qquad \psi_{Dt} \equiv \frac{\varepsilon}{\gamma} \frac{w_{Ds}}{1 - w_{As}} \left(\frac{P_{Dt}}{\widetilde{P}_t} \right)^{\gamma}.$$

This index is a direct generalization of Proposition 1 and, as before, is computable given expenditure shares w_{As}, w_{Ds}, price indices P_{At}, P_{Bt}, P_{Dt} and parameter values for ε and γ.

Similarly to Proposition 2, under Assumption 1 and appropriate choices for P_{At}, P_{Bt} and P_{Dt}, estimation reduces to only the two parameters ε and γ which are readily obtained from Equations (A.8) and (A.10).

Unlike the baseline framework, however, these preferences do not aggregate as easily. As shown in Alder, Boppart and Müller (2022, Proposition 2), aggregate expenditure shares over a measure N of households are now

$$\overline{w}_A = \frac{A(\boldsymbol{p})}{\bar{e}},$$

$$\overline{w}_B = \left(1 - \frac{A(\boldsymbol{p})}{\bar{e}}\right)\left[1 - \nu\left(\frac{B(\boldsymbol{p})}{\bar{e} - A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\right]\kappa,$$

$$\overline{w}_D = \left(1 - \frac{A(\boldsymbol{p})}{\bar{e}}\right)\nu\left(\frac{B(\boldsymbol{p})}{\bar{e} - A(\boldsymbol{p})}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma}\kappa.$$

where

$$\kappa \equiv \frac{1}{N}\int_0^N \left(\frac{e_h - A(\boldsymbol{p})}{\bar{e} - A(\boldsymbol{p})}\right)^{1-\varepsilon} dh.$$

Unlike the PIGL case, there is no representative level of expenditure in Muellbauer's (1975, 1976) sense, even though a representative agent exists.[17] Therefore, it is not possible to bake in the parameter κ into some representative level of expenditure and proceed as for an individual household. Instead, the expenditure function of the representative agent is now

$$c(u^{RA}, \boldsymbol{p}) = \left[1 + \varepsilon\left(u^{RA} + \frac{\nu\,\kappa}{\gamma}\left\{\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right\}\right)\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}) + A(\boldsymbol{p}).$$

Similar steps as before gives an aggregate price index of the same form as above, $P_t = (1 - \overline{w}_{As})P_{Dt}^{\frac{\gamma\phi_t}{\varepsilon}} P_{Bt}^{1-\frac{\gamma\phi_t}{\varepsilon}} + \overline{w}_{As}P_{At}$, but with weights now given by

$$\psi_{Bt} \equiv \left(1 - \frac{\varepsilon}{\gamma}\frac{\kappa_t}{\kappa_s}\frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}}\right)\left(\frac{P_{Bt}}{\widetilde{P}_t}\right)^{\gamma} \qquad \text{and} \qquad \psi_{Dt} \equiv \frac{\varepsilon}{\gamma}\frac{\kappa_t}{\kappa_s}\frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}}\left(\frac{P_{Dt}}{\widetilde{P}_t}\right)^{\gamma}.$$

where

$$\widetilde{P}_t \equiv \left[\left(1 - \frac{\varepsilon}{\gamma}\frac{\kappa_t}{\kappa_s}\frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}}\right)P_{Bt}^{\gamma} + \frac{\varepsilon}{\gamma}\frac{\kappa_t}{\kappa_s}\frac{\overline{w}_{Ds}}{1 - \overline{w}_{As}}P_{Dt}^{\gamma}\right]^{\frac{1}{\gamma}}.$$

Thus, to compute aggregate price indices, we either need to know the inequality measures κ in the time periods considered, or we need to impose the (rather strong) assumption that these measures remain constant over time for *all* groups considered.

[17] The expenditure level e^{RA} that induces the average expenditure shares for A and D are given by $e^{RA} = \bar{e}$ and $\left(1 - \frac{A(\boldsymbol{p})}{e^{RA}}\right)\left(e^{RA} - A(\boldsymbol{p})\right)^{-\varepsilon} = \left(1 - \frac{A(\boldsymbol{p})}{\bar{e}}\right)\left(\bar{e} - A(\boldsymbol{p})\right)^{-\varepsilon}\kappa$, respectively, and these generally differ.

Appendix B Additional Figures and Tables

Table B.1. CEX-CPI crosswalk.

	CEX category	CPI name	CPI code
1	Food at home	Food at home	SAF11
2	Food away from home	Food away from home	SEFV
3	Alcoholic beverages	Alcoholic beverages	SAF116
4	Rented dwellings	Rent of primary residence	SEHA
5	Owned dwellings[a]	Owners' equivalent rent of primary residence	SEHC
6	Other lodging	Lodging while out of town[b]	MUUR0000SE2102
		Lodging away from home[b]	SEHB
7	Utilities	Household energy	SAH21
8	Water	Water and sewerage maintenance	SEHG01
9	Phone	Communication	SAE2
10	Household O&F[c]	Household furnishings and operations	SAH3
11	Apparel	Apparel	SAA
12	Gasoline	Motor fuel	SETB
13	Other vehicle expenses	Motor vehicle maintenance and repair	SETD
		Motor vehicle insurance	SETE
		Motor vehicle fees	SETF
14	Public transportation	Public transportation	SETG
15	Health	Medical care	SAM
16	Entertainment	Recreation	SAR
17	Personal care	Personal care	SAG1
18	Reading	Recreational reading materials	SERG
19	Education	Education and communcation	SAE
20	Tobacco	Tobacco and smoking products	SEGA
21	Other expenses	Miscellaneous personal services	SEGD

Notes. The CEX categories follow the hierarchical groupings defined by the BLS. CPI are non-seasonally adjusted nationwide data for urban consumers.
[a] Rental equivalence value of owned dwellings as reported by the households.
[b] "Lodging away from home" from 1995–1997 and "lodging while out of town" afterwards.
[c] Operations and furnishing, includes "household operations", "housekeeping supplies" and "household furnishings and equipment".

Table B.2. Estimated Engel curve slopes and classification of expenditure categories.

Classification	Category	(1)	(2)	(3)	(4)	(5)	(6)
Luxuries	Owned dwellings	1.757	1.629	1.631	1.707	0.0001800	9.461
		(0.019)	(0.020)	(0.020)	(0.022)	(0.0000072)	(0.131)
	Household operations and furnishings	0.690	0.725	0.724	0.740	0.0001470	4.762
		(0.010)	(0.010)	(0.010)	(0.011)	(0.0000041)	(0.084)
	Entertainment	0.386	0.405	0.404	0.417	0.0000604	2.427
		(0.006)	(0.006)	(0.006)	(0.007)	(0.0000024)	(0.048)
	Other lodging	0.303	0.294	0.294	0.300	0.0000532	1.838
		(0.005)	(0.004)	(0.004)	(0.005)	(0.0000017)	(0.033)
	Food away from home	0.299	0.323	0.324	0.348	0.0000415	1.957
		(0.005)	(0.005)	(0.005)	(0.006)	(0.0000016)	(0.035)
	Education	0.280	0.281	0.281	0.286	0.0000560	1.825
		(0.006)	(0.006)	(0.006)	(0.007)	(0.0000023)	(0.048)
	Health	0.215	0.163	0.166	0.230	0.0000176	1.265
		(0.007)	(0.007)	(0.007)	(0.008)	(0.0000015)	(0.044)
	Public transport	0.137	0.139	0.139	0.120	0.0000197	0.692
		(0.003)	(0.003)	(0.003)	(0.003)	(0.0000008)	(0.019)
	Other vehicle expenditures	0.124	0.135	0.132	0.153	0.0000056	0.863
		(0.006)	(0.006)	(0.006)	(0.007)	(0.0000011)	(0.039)
	Other expenses	0.105	0.102	0.104	0.111	0.0000183	0.682
		(0.002)	(0.002)	(0.002)	(0.002)	(0.0000006)	(0.014)
	Apparel	0.098	0.122	0.118	0.116	0.0000205	0.689
		(0.004)	(0.004)	(0.004)	(0.004)	(0.0000015)	(0.028)
	Alcoholic beverages	0.044	0.047	0.047	0.046	0.0000061	0.255
		(0.001)	(0.001)	(0.001)	(0.001)	(0.0000003)	(0.008)
	Personal care	0.031	0.032	0.031	0.031	0.0000038	0.170
		(0.001)	(0.001)	(0.001)	(0.001)	(0.0000002)	(0.006)
	Reading	0.028	0.027	0.027	0.026	0.0000027	0.139
		(0.001)	(0.001)	(0.001)	(0.001)	(0.0000001)	(0.003)
Necessities	Water	−0.045	−0.051	−0.050	−0.037	−0.0000064	−0.220
		(0.002)	(0.002)	(0.002)	(0.002)	(0.0000002)	(0.010)
	Tobacco	−0.158	−0.166	−0.167	−0.146	−0.0000194	−0.843
		(0.003)	(0.003)	(0.003)	(0.003)	(0.0000007)	(0.023)
	Phone	−0.186	−0.186	−0.186	−0.182	−0.0000279	−1.065
		(0.003)	(0.003)	(0.003)	(0.003)	(0.0000007)	(0.017)
	Gasoline	−0.219	−0.216	−0.214	−0.168	−0.0000335	−0.958
		(0.004)	(0.004)	(0.004)	(0.004)	(0.0000009)	(0.027)
	Utilities	−0.362	−0.380	−0.381	−0.329	−0.0000445	−1.904
		(0.004)	(0.004)	(0.004)	(0.004)	(0.0000012)	(0.025)
	Rented dwellings	−1.496	−1.390	−1.387	−1.756	−0.0002220	−10.070
		(0.018)	(0.018)	(0.018)	(0.020)	(0.0000063)	(0.115)
	Food at home	−2.031	−2.035	−2.037	−2.013	−0.0002780	−11.970
		(0.012)	(0.012)	(0.012)	(0.014)	(0.0000066)	(0.082)
Category dummies		✓	✓	✓	✓	✓	✓
5-year age-bin fixed effects			✓	✓	✓	✓	✓
Year fixed effects				✓	✓	✓	✓
State fixed effects					✓	✓	✓
Adjusted R^2		0.579	0.591	0.597	0.605	0.584	0.606
Observations		1,562,211	1,542,765	1,542,765	1,351,413	1,351,413	1,351,413

Notes. Standard errors clustered at the household level in parentheses. Columns (1) to (4) show the coefficient estimates from a weighted least square regression of expenditure shares (in percent) on the expenditure decile interacted with expenditure category dummies using the CEX household sampling weights. All fixed effects are by expenditure category. Columns (5) and (6) show coefficient estimates for the same regression but using the expenditure level and log expenditure level instead of the expenditure decile.

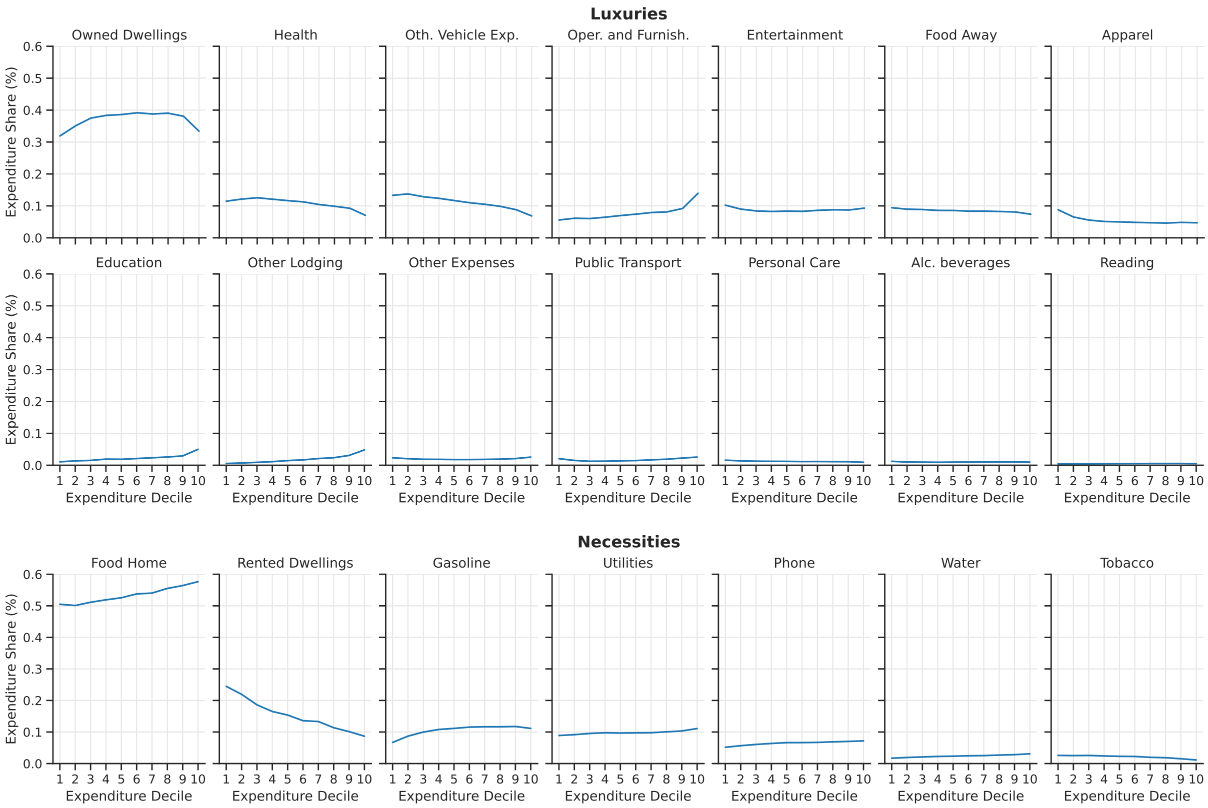

Figure B.1. Expenditure shares within luxuries and necessities by expenditure decile.

Notes. The figure shows the expenditure share of each expenditure category within the group of luxuries and necessities averaged over all years. If quasi separability held perfectly, the expenditure share would be constant for all expenditure deciles. Thus, any slope different from zero highlights the residual nonhomotheticity within luxuries and necessities.

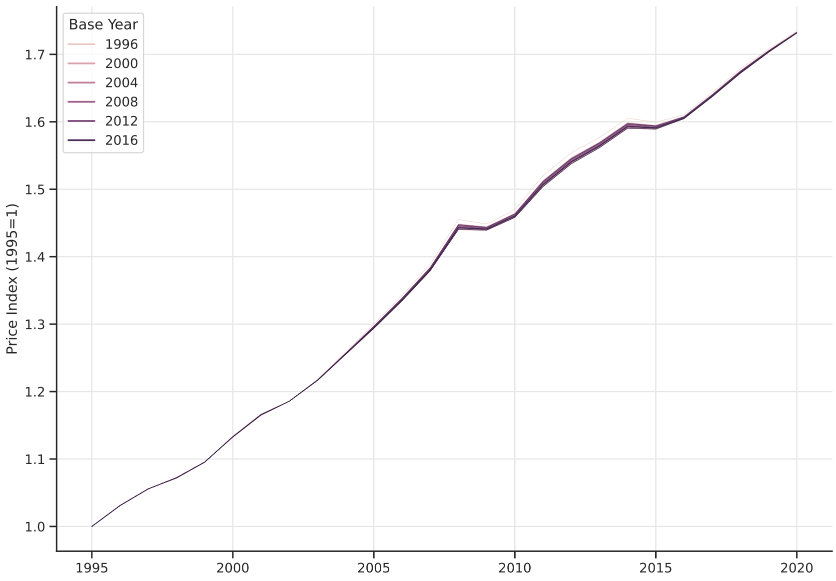

Figure B.2. PIGL representative agent Generalized Sato-Vartia price index for different base years.

Notes. The price index is calculated under quasi separability. Each line represents the representative agent price index for a different base year, but normalized to one in 1995.

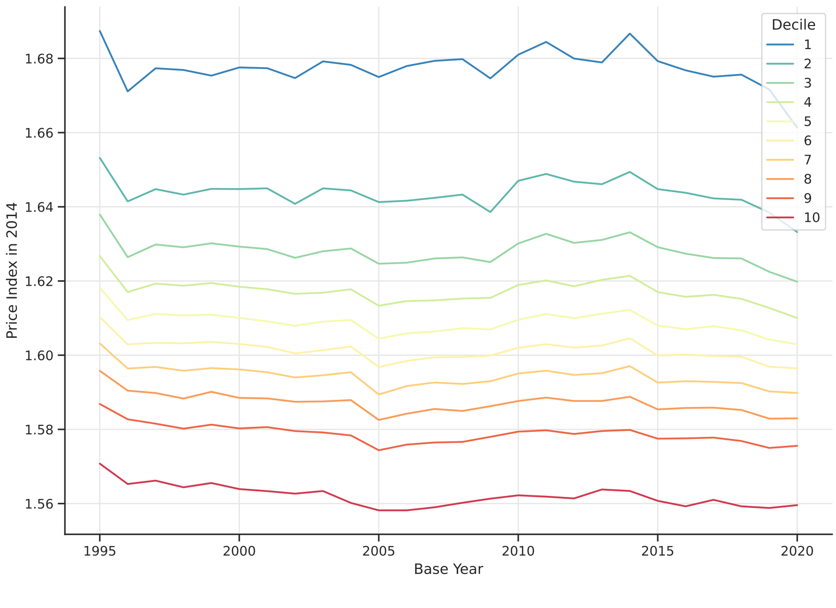

Figure B.3. Generalized Sato-Vartia price index in 2014 by expenditure decile for different base years.

Notes. The price index is calculated under quasi separability. The horizontal axis describes the base year of the price index and the vertical axis the respective value of the price index in 2014. Price indices are all normalized to one in 1995. The price index for each expenditure decile is calculated as the price index of the PIGL representative agent over households within each respective decile.

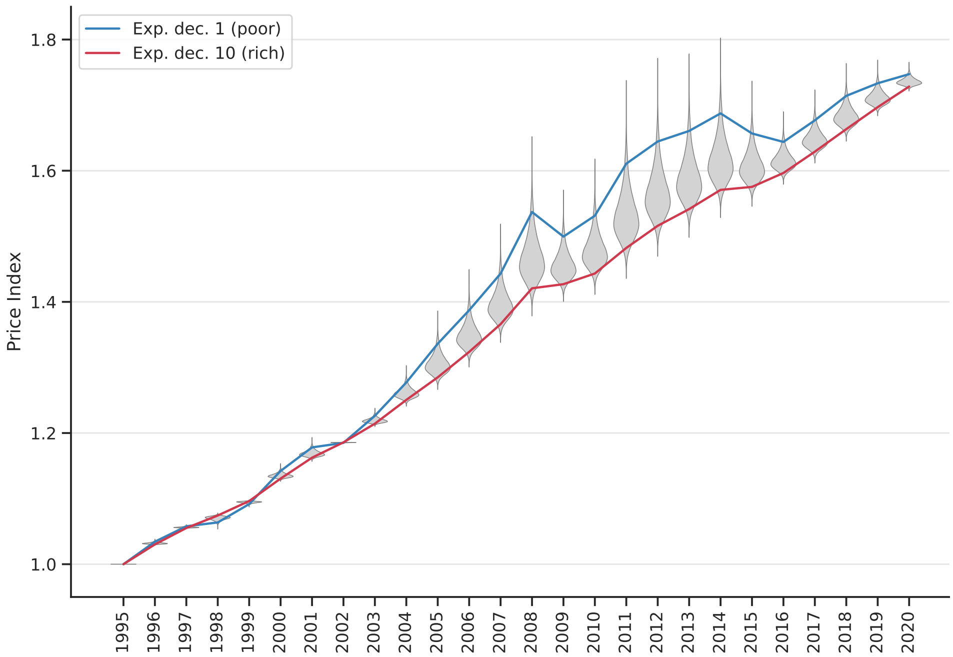

Figure B.4. Distribution of the Generalized Sato-Vartia price index under quasi separability.

Notes. The gray shaded area shows the kernel density estimate of the distribution of Generalized Sato-Vartia price indices. The index is calculated for each household in the sample of 1995. The blue line shows the PIGL RA price index for the poorest 10 percent. The red line shows the PIGL RA price index for the richest 10 percent.

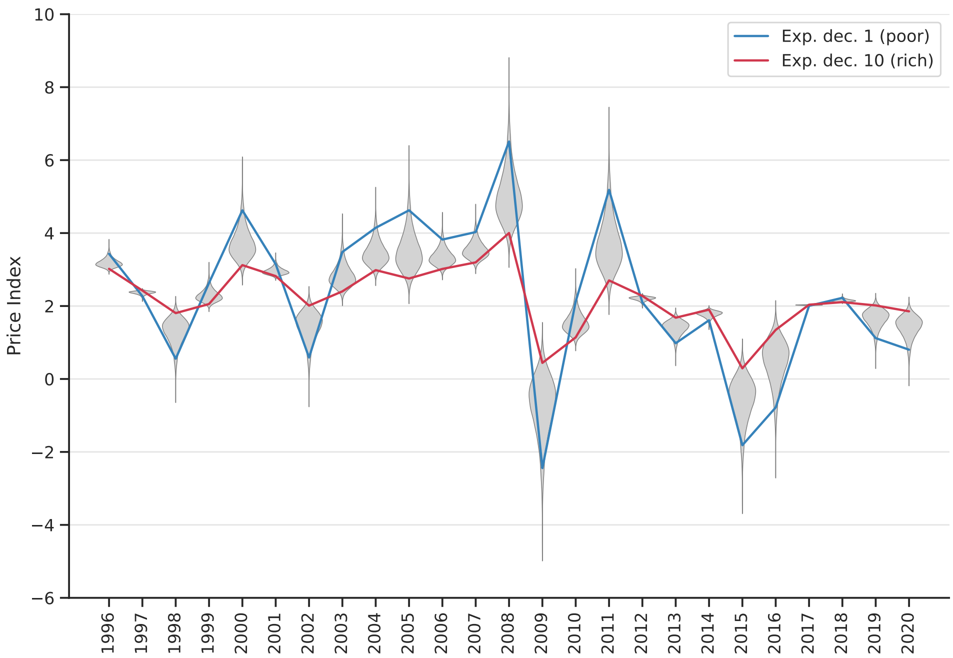

Figure B.5. Distribution of Generalized Sato-Vartia inflation under quasi separability.

Notes. The gray shaded area shows the kernel density estimate of the distribution of inflation rates. The inflation is calculated for each household in the sample of 1995. The blue line shows the PIGL RA inflation for the poorest 10 percent. The red line shows the PIGL RA inflation for the richest 10 percent.

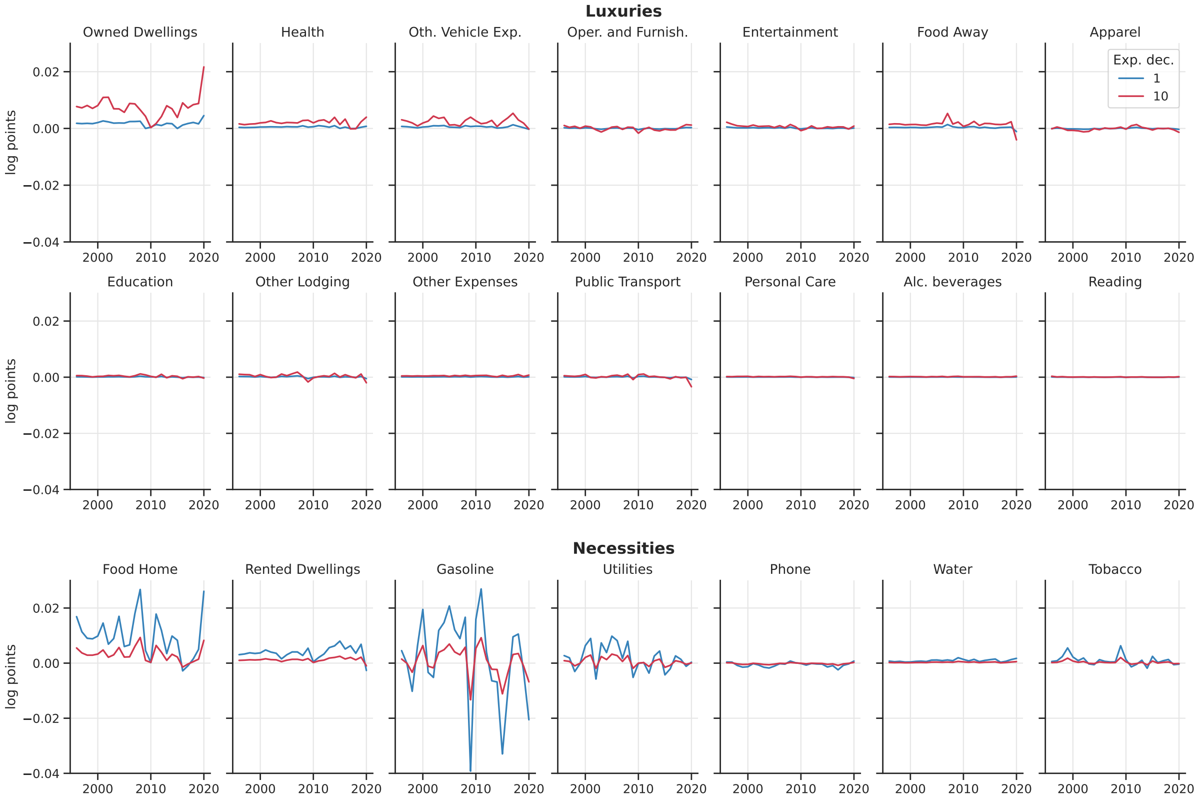

Figure B.6. Inflation decomposition by expenditure categories.

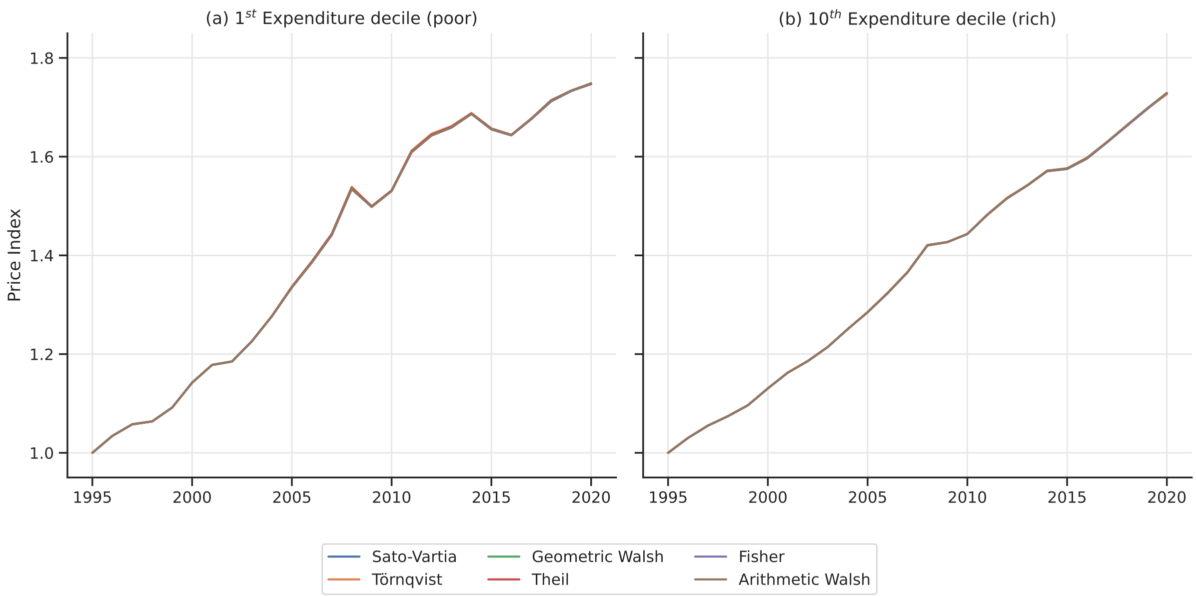

Figure B.7. Comparison of different generalized superlative price indices.

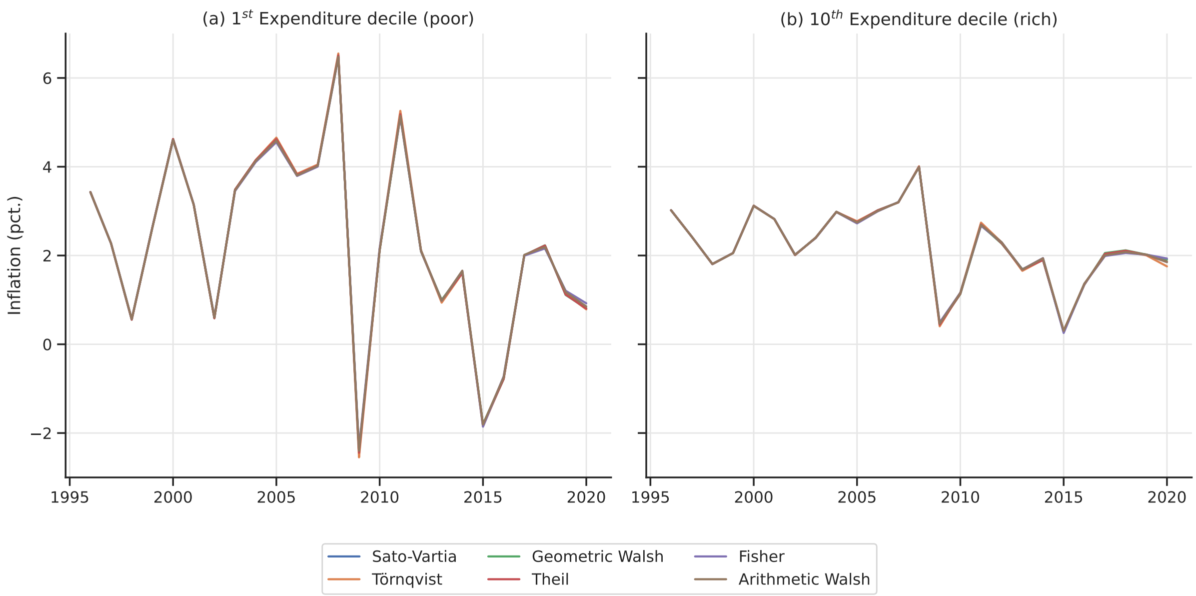

Figure B.8. Comparison of inflation across different generalized superlative indices.

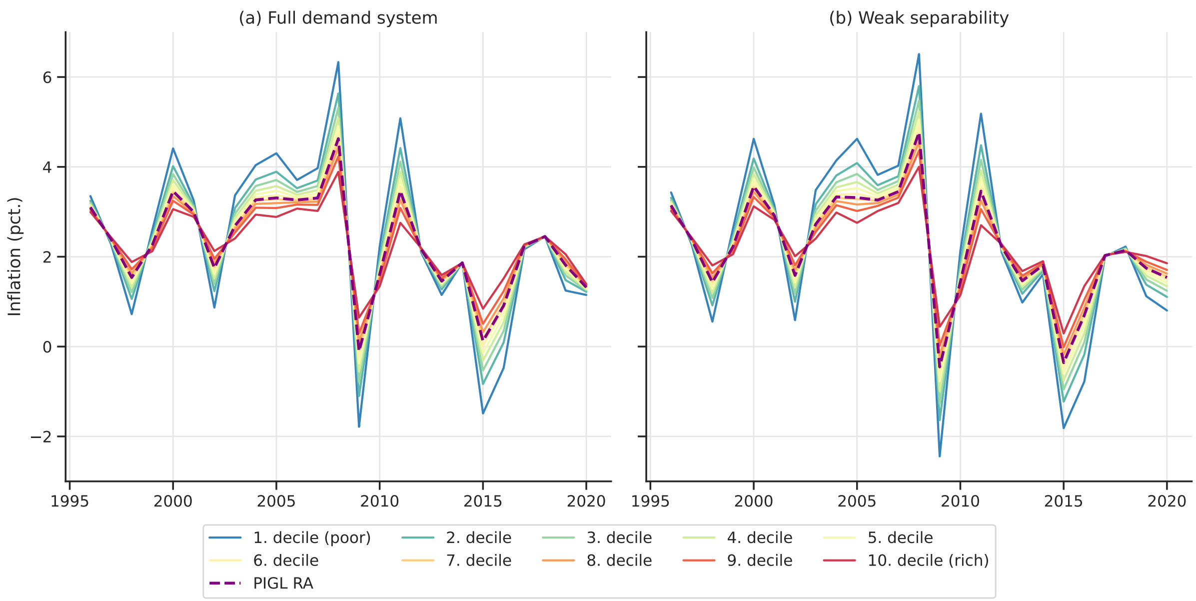

Figure B.9. Comparison of Generalized Sato-Vartia inflation between the full demand system estimation and under quasi separability.